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SELECT MECHANICAL EXERCISES.

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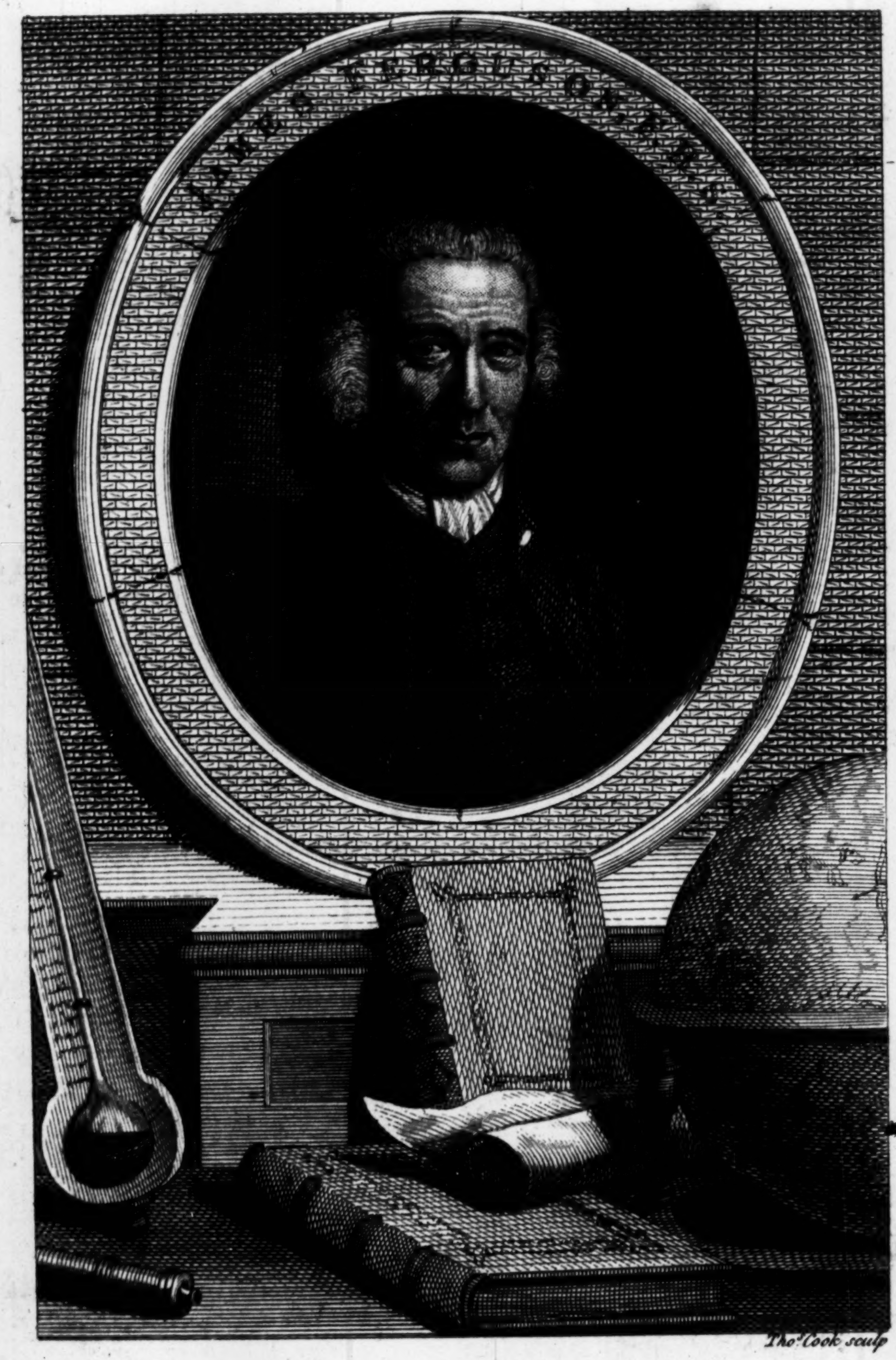
JAMES FERGUSON, F.R.S.

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SELECT
Mechanical Exerciſes:

Shewing how to conſtruct different
CLOCKS, ORRERIES, and SUN-DIALS,
ON PLAIN AND EASY PRINCIPLES.

WITH SEVERAL
MISCELLANEOUS ARTICLES;

AND
NEW TABLES,

I. For expeditiouſly computing the Time of any NEW or FULL MOON within the Limits of 6000 Years before and after the 18th Century.

II. For graduating and examining the uſual Lines on the SECTOR, PLAIN SCALE, and GUNTER.

Illuſtrated with COPPER-PLATES.

To which is prefixed,
A ſhort Account of the Life of the Author.

By JAMES FERGUSON, F. R. S.

THE SECOND EDITION.

LONDON:
Printed for W. Strahan; and T. Cadell, in the Strand.
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A SHORT

ACCOUNT

OF THE

LIFE OF THE AUTHOR.

AS this is probably the laſt Book I ſhall ever publiſh, I beg leave to prefix to it a ſhort account of myſelf, and of the manner I firſt began, and have ſince proſecuted my ſtudies. For, as my ſetting out in life from a very low ſtation, and in a remote part of the Iſland, has occaſioned ſome falſe and indeed very improbable particulars to be related of me, I therefore think it the better way, inſtead of contradicting them one by one, to give a faithful and circumſtantial detail of my whole proceedings, from my firſt obſcure beginning to the preſent time: wherein, if I ſhould inſert ſome particulars of little moment, I hope the good-natured Reader will kindly excuſe me.

I was born in the year 1710, a few miles from Keith, a little village in Bamffshire, in the North of Scotland; and can with pleaſure ſay, that my parents, though poor, were religious and honeſt; lived in good repute with all who knew them, and died with good characters.

As my father had nothing to ſupport a large family but his daily labour, and the profits ariſing from a few acres of land which he rented, it was not to be expected that he could beſtow much on the education of his children: yet they were not neglected; for, at his leiſure hours, he taught them to read and write. And it was while he was teaching my elder brother to read the Scotch Catechiſm that I acquired my reading.

Aſhamed

BRITISH MUSEUM JAN

Aſhamed to aſk my father to inſtruct me, I uſed, when he and my brother were abroad, to take the Catechiſm, and ſtudy the leſſon which he had been teaching my brother: and when any difficulty occurred, I went to a neighbouring old woman, who gave me ſuch help as enabled me to read tolerably well before my father had thought of teaching me.

Some time after, he was agreeably ſurpriſed to find me reading by myſelf: he thereupon gave me further inſtruction, and alſo taught me to write; which, with about three months I afterward had at the grammar-ſchool at Keith, was all the education I ever received.

My taſte for mechanics aroſe from an odd accident.—When about 7 or 8 years of age, a part of the roof of the houſe being decayed, my father, deſirous of mending it, applied a prop and lever to an upright ſpar to raiſe it to its former ſituation; and, to my great aſtoniſhment, I ſaw him, without conſidering the reaſon, lift up the ponderous roof as if it had been a ſmall weight. I attributed this at firſt to a degree of ſtrength that excited my terror as well as wonder: but thinking further of the matter, I recollected that he had applied his ſtrength to that end of the lever which was furtheſt from the prop; and finding, on enquiry, that this was the means whereby the ſeeming wonder was effected, I begun making levers (which I then called bars); and by applying weights to them different ways, I found the power gained by my bar was juſt in proportion to the lengths of the different parts of the bar on either ſide of the prop.—I then thought it was great pity that, by means of this bar, a weight could be raiſed but a very little way. On this, I ſoon imagined, that, by pulling round a wheel, the weight might be raiſed to any height by tying a rope to the weight, and winding the rope round the axle of the wheel; and that the power gained muſt be juſt as great as the wheel was broader than the axle was thick; and found it to be exactly ſo, by hanging one weight to a rope put round the wheel, and another to the rope that coiled round the axle. So that, in theſe two machines, it appeared very plain, that their advantage was as great as the ſpace gone thro' by the working power exceeded the ſpace gone through by the weight: and this property I alſo thought muſt take place in a wedge for cleaving wood; but then, I happened not to think of the ſcrew.—By means of a turning lathe which my father had, and ſometimes uſed, and a little knife, I was enabled to make wheels and other things neceſſary for my purpoſe.

I then wrote a ſhort account of theſe machines, and ſketched out figures of them with a pen, imagining it to be the firſt treatiſe of the kind that ever was written; but found my miſtake when I afterward ſhewed it to a gentleman, who told me that theſe things were known long before, and ſhewed me a printed book in which they were treated of; and I was much pleaſed when I found, that my account (ſo far as I had carried it) agreed with the principles of mechanics in the book he ſhewed me. And from that time my mind preſerved a conſtant tendency to improve in that ſcience.

But, as my father could not afford to maintain me while I was in purſuit only of theſe matters, and I was rather too young and weak for hard labour, he put me out to a neighbour to keep ſheep, which I continued to do for ſome years; and in that time I began to ſtudy the ſtars in the night. In the day-time I amuſed myſelf by making models of mills, ſpinning-wheels, and ſuch other things as I happened to ſee.

I then went to ſerve a conſiderable farmer in the neighbourhood, whoſe name was James Glaſhan. I found him very kind and indulgent; but he ſoon obſerved, that in the evenings, when my work was over, I went into a field with a blanket about me; lay down on my back, and ſtretched a thread with ſmall beads upon it, at arms' length, between my eye and the ſtars; ſliding the beads upon it till they hid ſuch and ſuch ſtars from my eye, in order to take their apparent diſtances from one another; and then, laying the thread down on a paper, I marked the ſtars thereon by the beads, according to their reſpective poſitions, having a candle by me. My maſter at firſt laughed at me; but, when I explained my meaning to him, he encouraged me to go on: and that I might make fair copies in the daytime of what I had done in the night, he often worked for me himſelf. I ſhall always have a reſpect for the memory of that man.

One day he happened to ſend me with a meſſage to the Reverend Mr.

John Gilchriſt, miniſter at Keith, to whom I had been known from my childhood. I carried my ſtar-papers to ſhew them to him, and found him looking over a large parcel of maps, which I ſurveyed with great pleaſure, as they were the firſt I had ever ſeen. He then told me that the Earth is round like a ball, and explained the map of it to me. I requeſted him to lend me *that* map, to take a copy of it in the evenings. He chearfully conſented to this, giving me at the ſame time a pair of compaſſes, a ruler, pens, ink, and paper; and diſmiſſed me with an injunction not to neglect my maſter's buſineſs by copying the map, which I might keep as long as I pleaſed.

For this pleaſant employment, my maſter gave me more time than I could reaſonably expect; and often took the threſhing-flail out of my hands, and worked himſelf, while I ſat by him in the barn, buſy with my compaſſes, ruler, and pen.

When I had finiſhed the copy, I aſked leave to carry home the map: he told me I was at liberty to do ſo, and might ſtay two hours to converſe with the miniſter.—In my way thither, I happened to paſs by the ſchool at which I had been before, and ſaw a genteel-looking man (whoſe name I afterwards learnt was Cantley) painting a ſun-dial on the wall. I ſtopt a while to obſerve him, and the ſchool-maſter came out, and aſked me what parcel it was that I had under my arm. I ſhewed him the map, and the copy I had made of it, wherewith he appeared to be very well pleaſed, and aſked me whether I ſhould not like to learn of Mr. Cantley to make ſun-dials. Mr. Cantley looked at the copy of the map, and commended it much; telling the ſchool-maſter (Mr. John Skinner) that it was a pity I did not meet with notice and encouragement. I had a good deal of converſation with him, and found him to be quite affable and communicative; which made me think I ſhould be extremely happy if I could be further acquainted with him.

I then proceeded with the map to the miniſter, and ſhewed him the copy of it.—While we were converſing together, a neighbouring gentleman, Thomas Grant, Eſq; of Achoynaney, happened to come in; and the miniſter immediately introduced me to him, ſhewing him what I had done. He expreſſed great ſatisfaction, aſked me ſome queſtions about the conſtruction of maps, and told me, that if I would go and live at his houſe, he would order his butler, Alexander Cantley, to give me a great deal of inſtruction. Finding that this Cantley was the man whom I had ſeen painting the ſun-dial, and of whom I had already conceived a very high opinion, I told 'Squire Grant, that I ſhould rejoice to be at his houſe as ſoon as the time was expired for which I was engaged with my preſent maſter.—He very politely offered to put one in my place; but this I declined.

When the term of my ſervitude was out, I left my good maſter, and went to the gentleman's houſe, where I quickly found myſelf with a moſt humane good family. Mr. Cantley the butler ſoon became my friend, and continued ſo till his death. He was the moſt extraordinary man that I ever was acquainted with, or perhaps ever ſhall ſee; for he was a complete maſter of arithmetic, a good mathematician, a maſter of muſick on every known inſtrument except the harp, underſtood Latin, French, and Greek, let blood extremely well, and could even preſcribe as a phyſician upon any urgent occaſion. He was what is generally called *ſelf-taught*; but, I think, he might with much greater propriety have been termed GOD ALMIGHTY's ſcholar.

He immediately began to teach me decimal arithmetic, and algebra; for

I had already learnt vulgar arithmetic, at my leiſure hours, from books. He then proceeded to teach me the elements of geometry; but, to my inexpreſſible grief, juſt as I was beginning that branch of ſcience, he left Mr. Grant, and went to the late Earl of Fife's, at ſeveral miles diſtance. The good family I was then with could not prevail with me to ſtay after he was gone; ſo I left them, and went to my father's.

He had made me a preſent of Gordon's Geographical Grammar, which, at that time, was to me a great treaſure. There is no figure of a globe in it, although it contains a tolerable deſcription of the globes, and their uſe. From this deſcription I made a globe in three weeks at my father's, having turned the ball thereof out of a piece of wood; which ball I covered with paper, and delineated a map of the world upon it; made the meridian ring and horizon of wood; covered them with paper, and graduated them; and was happy to find, that, by my globe (which was the firſt I ever ſaw) I could ſolve the problems.

But this was not likely to afford me bread, and I could not think of ſtaying with my father, who I knew full well could not maintain me in that way, as it would be of no ſervice to him; and he had, without my aſſiſtance, hands ſufficient for all his work.

I then went to a miller, thinking it would be a very eaſy buſineſs to attend the mill, and that I ſhould have a great deal of leiſure-time to ſtudy decimal arithmetic and geometry. But my maſter, being too fond of tipling at an ale-houſe, left the whole care of the mill to me, and almoſt ſtarved me for want of victuals; ſo that I was glad when I could have a little oat-meal mixed with cold water to eat. I was engaged for a year in this man's ſervice, at the end of which I left him, and returned in a very weak ſtate to my father's.

Soon after I had recovered my former ſtrength, a neighbouring farmer, who practiſed as a phyſician in that part of the country, came to my father's, wanting to have me as a labouring ſervant. My father adviſed me to go to Doctor Young, telling me that the Doctor would inſtruct me in that part of his buſineſs. This he promiſed to do, which was a temptation to me. But inſtead of performing his promiſe, he kept me conſtantly to very hard labour, and never once ſhewed me one of his books. All his ſervants complained that he was the hardeſt maſter they had ever lived with; and it was my misfortune to be engaged with him for half a year. But, at the end of three months, I was ſo much overwrought, that I was almoſt diſabled, which obliged me to leave him: and he was ſo unjuſt as to give me nothing at all for the time I had been with him, becauſe I did not complete my half-year's ſervice; though he knew that I was not able, and had ſeen me working for the laſt fortnight, as much as poſſible, with one hand and arm, when I could not lift the other from my ſide. And what I thought was particularly hard, he never once tried to give me the leaſt relief, further than once bleeding me, which rather did me hurt than good, as I was very weak, and much emaciated. I then went to my father's, where I was confined for two months on account of my hurt, and deſpaired of ever recovering the uſe of my left arm. And during all that time, the Doctor never once came to ſee me, although the diſtance was not quite two miles.—But my friend Mr. Cantley hearing of my misfortune, at twelve miles diſtance, ſent me proper medicines and applications, by means of which I recovered the uſe of my arm; but found myſelf too weak to think of going into ſervice again, and had entirely loſt my appetite, ſo that I could take nothing but a draught of milk once a-day, for many weeks.

In order to amuſe myſelf in this low ſtate, I made a wooden clock, the frame of which was alſo of wood; and it kept time pretty well. The bell, on which the hammer ſtruck the hours, was the neck of a broken bottle.

Having then no idea how any time-keeper could go but by a weight and a line, I wondered how a watch could go in all poſitions; and was ſorry that I had never thought of aſking Mr. Cantley, who could very eaſily have informed me. But happening one day to ſee a gentleman ride by my father's houſe (which was cloſe by a public road), I aſked him what o'clock it then was: he looked at his watch, and told me. As he did that with ſo much good-nature, I begged of him to ſhew me the inſide of his watch: and though he was an entire ſtranger, he immediately opened the watch, and put it into my hands. I ſaw the ſpring-box with part of the chain round it, and aſked him what it was that made the box turn round: he told me that it was turned round by a ſteel ſpring within it. Having then never ſeen any other ſpring than that of my father's gun-lock, I aſked how a ſpring within a box could turn the box ſo often round as to wind all the chain upon it. He anſwered, that the ſpring was long and thin; that one end of it was faſtened to the axis of the box, and the other end to the inſide of the box; that the axis was fixed, and the box was looſe upon it. I told him I did not yet thoroughly underſtand the matter: Well, my lad, ſays he, take a long thin piece of whalebone, hold one end of it faſt between your finger and thumb, and wind it round your finger: it will then endeavour to unwind itſelf; and if you fix the other end of it to the inſide of a ſmall hoop, and leave it to itſelf, it will turn the hoop round and round, and wind up a thread tied to the outſide of the hoop. —I thanked the gentleman, and told him that I underſtood the thing very well. I then tried to make a watch with wooden wheels, and made the ſpring of whalebone; but found that I could not make the watch go when the balance was put on, becauſe the teeth of the wheels were rather too weak to bear the force of a ſpring ſufficient to move the balance; altho' the wheels would run faſt enough when the balance was taken off. I incloſed the whole in a wooden caſe, very little bigger than a breakfaſt tea-cup: but a clumſy neighbour one day looking at my watch, happened to let it fall; and turning haſtily about to pick it up, ſet his foot upon it, and cruſhed it all to pieces; which ſo provoked my father, that he was almoſt ready to beat the man; and diſcouraged me ſo much, that I never attempted to make ſuch another machine again, eſpecially as I was thoroughly convinced I could never make one that would be of any real uſe.

As ſoon as I was able to go abroad, I carried my globe, clock, and copies of ſome other maps beſides that of the world, to the late Sir James Dunbar of Durn (about ſeven miles from where my father lived), as I had heard that Sir James was a very good-natur'd, friendly, inquiſitive gentleman. He received me in a very kind manner, was pleaſed with what I ſhewed him, and deſired I would clean his clocks. This, for the firſt time, I attempted; and then begun to pick up ſome money in that way about the country, making Sir James's houſe my home, at his deſire.

Two large globular ſtones ſtood on the top of his gate: on one of them I painted (with oil colours) a map of the terreſtrial globe, and on the other a map of the celeſtial, from a planiſphere of the ſtars which I copied on

paper

paper from a celeſtial globe belonging to a neighbouring gentleman. The poles of the painted globes ſtood toward the poles of the heavens; on each, the 24 hours were placed around the equinoctial, ſo as to ſhew the time of the day when the ſun ſhone out, by the boundary where the half of the globe at any time enlightened by the ſun was parted from the other half in the ſhade; the enlightened parts of the terreſtrial globe anſwering to the like enlightened parts of the earth at all times. So that, whenever the ſun ſhone on the globe, one might ſee to what places the ſun was then riſing, to what places it was ſetting, and all the places where it was then day or night, throughout the earth.

During the time I was at Sir James's hoſpitable houſe, his ſiſter, the Honourable the Lady Dipple, came there on a viſit, and Sir James introduced me to her. She aſked me whether I could draw patterns for needle-work on aprons and gowns. On ſhewing me ſome, I undertook the work, and drew ſeveral for her; ſome of which were copied from her patterns, and the reſt I did according to my own fancy. On this, I was ſent for by other ladies in the country, and begun to think myſelf growing very rich by the money I got for ſuch drawings; out of which I had the pleaſure of occaſionally ſupplying the wants of my poor father.

Yet all this while I could not leave off ſtar-gazing in the nights, and taking the places of the planets among the ſtars by my above-mentioned thread. By this I could obſerve how the planets changed their places among the ſtars, and delineated their paths on the celeſtial map, which I had copied from the above-mentioned celeſtial globe.

By obſerving what conſtellations the Ecliptick paſſed through in that map, and comparing theſe with the ſtarry heaven, I was ſo impreſs'd as ſometimes to imagine that I ſaw the Ecliptic in the heaven, among the ſtars, like a broad circular road for the ſun's apparent courſe; and fancied the paths of the planets to reſemble the narrow ruts made by cart-wheels, ſometimes on one ſide of a plain road and ſometimes on the other, croſſing the road at ſmall angles, but never going far from either ſide of it.

Sir James's houſe was full of pictures and prints, ſeveral of which I copied with pen and ink: this made him think I might become a painter.

Lady Dipple had been but a few weeks there, when William Baird, Eſq; of Auchmedden, came on a viſit: he was the huſband of one of that lady's daughters, and I found him to be very ingenious and communicative; he invited me to go to his houſe and ſtay ſome time with him, telling me that I ſhould have free acceſs to his library, which was a very large one; and that he would furniſh me with all ſorts of implements for drawing. I went thither, and ſtaid about eight months; but was much diſappointed in finding no books of aſtronomy in his library, except what was in the two volumes of Harris's Lexicon Technicum, altho' there were many books on geography and other ſciences: ſeveral of theſe indeed were in Latin, and more in French; which being languages that I did not underſtand, I had recourſe to him for what I wanted to know of theſe ſubjects, which he chearfully read to me; and it was as eaſy for him, at ſight, to read Engliſh from a Greek, Latin, or French book, as from an Engliſh one. He furniſhed me with pencils and Indian ink, ſhewing me how to draw with them: and although he had but an indifferent hand at that work, yet he was a very acute judge; and conſequently a very fit perſon for ſhewing me how to correct my own work. He was the firſt who ever ſat to me for a picture, and I found it was much eaſier to draw from the life than from any picture whatever, as nature was more ſtriking than any imitation of it.

Lady Dipple came to his houſe in about half a year after I went thither. And as they thought I had a genius for painting, they conſulted together about what might be the beſt way to put me forward. Mr. Baird thought it would be no difficult matter to make a collection for me among the neighbouring gentlemen, to put me to a painter at Edinburgh: but he found, upon trial, that nothing worth the while could be done among them. And as to himſelf, he could not do much that way, becauſe he had but a ſmall eſtate, and a very numerous family.

Lady Dipple then told me that ſhe was to go to Edinburgh next ſpring, and that if I would go thither, ſhe would give me a year's bed and board at her houſe *gratis*, and make all the intereſt ſhe could for me among her acquaintance there.—I thankfully accepted of her kind offer; and inſtead of giving me one year, ſhe gave me two. I carried with me a letter of recommendation from the Lord Pitſligo (a near neighbour of 'Squire Baird's) to Mr. John Alexander, a painter in Edinburgh; who allowed me to paſs an hour every day at his houſe, for a month, to copy from his drawings; and ſaid he would teach me to paint in oil-colours, if I would ſerve him ſeven years, and my friends would maintain me all that time: but this was too much for me to deſire them to do; nor did I chuſe to ſerve ſo long. I was then recommended to other painters, but they would do nothing without money. So I was quite at a loſs what to do.

In a few days after this, I received a letter of recommendation from my good friend 'Squire Baird to the Reverend

rend Dr. Robert Keith at Edinburgh, to whom I gave an account of my bad ſucceſs among the painters there. He told me, that if I would copy from nature, I might do without their aſſiſtance; as all the rules for drawing ſignified but very little when one came to draw from the life: and, by what he had ſeen of my drawings brought from the North, he judged I might ſucceed very well in drawing pictures from the life, in Indian ink, on vellum. He then ſat to me for his own picture, and ſent me with it and a letter of recommendation to the Right Honourable the Lady Jane Douglas, who lived with her mother, the Marchioneſs of Douglas, at Merchiſton-houſe, near Edinburgh. Both the Marchioneſs and Lady Jane behaved to me in the moſt friendly manner, on Dr. Keith's account, and ſat for their pictures; telling me at the ſame time, that I was in the very room in which Lord Napier invented and computed the Logarithms; and that, if I thought it would inſpire me, I ſhould always have the ſame room whenever I came to Merchiſton.—I ſtaid there ſeveral days, and drew ſeveral pictures of Lady Jane; of whom it was hard to ſay, whether the greatneſs of her beauty, or the goodneſs of her temper and diſpoſitions, was the moſt predominant. She ſent theſe pictures to adies of her acquaintance, in order to recommend me to them; by which means I ſoon had as much buſineſs as I could poſſibly manage, ſo as not only to put a good deal of money in my own pocket, but alſo to ſpare what was ſufficient to help to ſupply my father and mother in their old age.—Thus a buſineſs was providentially put into my hands, which I followed for ſix and twenty years.

Lady Dipple, being a woman of the ſtricteſt piety, kept a watchful eye over me at firſt, and made me give her an exact account at night of what families I had been in throughout the day, and of the money I had received. She took the money each night, deſiring I would keep an account of what I had put into her hands; telling me that I ſhould duly have, out of it, what I wanted for clothes, and to ſend to my father.—But, in leſs than half a year, ſhe told me that ſhe would thenceforth truſt me with being my own banker; for ſhe had made a good deal of private enquiry how I had behaved when I was out of her ſight through the day; and was ſatisfied with my conduct.

During my two years' ſtay at Edinburgh, I ſomehow took a violent inclination to ſtudy anatomy, ſurgery, and phyſic, all from reading of books, and converſing with gentlemen, on theſe ſubjects; which, for that time, put all thoughts of aſtronomy out of my mind, and I had no inclination to become acquainted with any one there who taught either mathematics or aſtronomy: for nothing would ſerve me but to be a Doctor.

At the end of the ſecond year I left Edinburgh, and went to ſee my father, thinking myſelf tolerably well qualified to be a phyſician in that part of the country; and I carried a good deal of medicines, plaiſters, *&c.* thither. — But to my mortification, I ſoon found that all my medical theories and ſtudy were of little uſe in practice. And then, finding that very few paid me for the medicines they had, and that I was far from being ſo ſucceſsful as I could wiſh, I quite left off that buſineſs, and began to think of taking to the more ſure one of drawing pictures again.—For this purpoſe I went to Inverneſs, where I had eight months buſineſs.

When I was there, I began to think of aſtronomy again; and was heartily ſorry for having quite neglected it at Edinburgh, where I might have improved my knowledge by converſing with thoſe who were very able to aſſiſt me.—I began to compare the Ecliptic with its twelve ſigns (through which the ſun goes in twelve months) to the circle of 12 hours on the dial-plate of a watch, the hour-hand to the Sun, and the minute-hand to the Moon, moving in the Ecliptic; the one always overtaking the other at a place forwarder than it did at their laſt conjunction before. On this, I contrived and finiſhed a ſcheme on paper for ſhewing the motions and places of the Sun and Moon in the Ecliptic on each day of the year, perpetually; and conſequently the days of all the New and Full Moons.

To this I wanted to add a method for ſhewing the Eclipſes of the Sun and Moon; of which I knew the cauſe long before, by having obſerved that the Moon was, for one half of her period, on the North ſide of the Ecliptic, and for the other half on the South. But, having not obſerved her courſe long enough among the Stars by my above-mentioned thread, ſo as to delineate her path upon my celeſtial map, in order to find the two oppoſite points of the Ecliptic in which her orbit croſſes it, I was altogether at a loſs how and where in the Ecliptic (in my ſcheme) to place theſe interſecting points: this was in the year 1739.

At laſt, I recollected, that when I was with 'Squire Grant of Achoynaney in the year 1730, I had read, that on the 1ſt of January 1690, the Moon's aſcending Node was on the 10th minute of the firſt degree of Aries; and that her Nodes moved backward thro' the whole Ecliptic in 18 years and 224 days, which was at the rate of 3 min. 11 ſec. every 24 hours. But, as I ſcarce knew in the year 1730 what the Moon's Nodes meant, I took no further notice of it at that time.

However, in the year 1739, I ſet to work at Inverneſs; and after a tedious calculation of the ſlow motion of the

Nodes from Jan. 1690 to Jan. 1740, it appeared to me, that (if I was ſure I had remembered right) the Moon's aſcending Node muſt be in 23 deg. 25 min. of Cancer at the beginning of the year 1740. And ſo I added the Eclipſe-part to my ſcheme, and called it *The Aſtronomical Rotula*.

When I had finiſhed it, I ſhewed it to the Reverend Mr. Alexander MacBean, one of the miniſters at Inverneſs, who told me he had a ſet of almanacks by him for ſeveral years paſt, and would examine it by the Eclipſes mentioned in them. We examined it together, and found that it agreed throughout with the days of all the New and Full Moons and Eclipſes mentioned in theſe almanacks; which made me think I had conſtructed it upon true aſtronomical principles. On this, Mr. Macbean deſired me to write to Mr. Maclaurin, profeſſor of the mathematics at Edinburgh, and give him an account of the methods by which I had formed my plan, requeſting him to correct it where it was wrong. He returned me a moſt polite and friendly anſwer (although I had never ſeen him during my ſtay at Edinburgh) and informed me that I had only miſtaken the radical mean place of the aſcending Node by a quarter of a degree; and that, if I would ſend the drawing of my Rotula to him, he would examine it, and endeavour to procure me a ſubſcription to defray the charges of engraving it on copper-plates, if I choſe to publiſh it. I then made a new and correct drawing of it, and ſent it to him, who ſoon got me a very handſome ſubſcription by ſetting the example himſelf, and ſending ſubſcription-papers to others.

I then returned to Edinburgh, and had the Rotula-plates engraved there by Mr. Cooper*. It has gone through ſeveral impreſſions, and always ſold very well till the year 1752, when the ſtile was changed, which rendered it quite uſeleſs.—Mr. Maclaurin received me with the greateſt civility when I firſt went to ſee him at Edinburgh. He then became an exceeding good friend to me, and continued ſo till his death.

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* Cooper was maſter to the juſtly celebrated Mr. Robert Strange, who was at that time his apprentice.

One day I requeſted him to ſhew me his Orrery, which he immediately did. I was greatly delighted with the motions of the Earth and Moon in it, and would gladly have ſeen the wheel-work, which was concealed in a braſs box, and the box and planets above it were ſurrounded by an armillary ſphere. But he told me, that he never had opened it; and I could eaſily perceive that it could not be opened but by the hand of ſome ingenious clock-maker, and not without a great deal of time and trouble.

After a good deal of thinking, and calculation, I found that I could contrive the wheel-work for turning the planets in ſuch a machine, and giving them their progreſſive motions; but ſhould be very well ſatisfied if I could make an Orrery to ſhew the motions of the Earth and Moon, and of the Sun round its axis. I then employed a turner to make me a ſufficient number of wheels and axles, according to patterns which I gave him in drawing: and after having cut the teeth in the wheels by a knife, and put the whole together, I found that it anſwered all my expectations. It ſhewed the Sun's motion round his axis, the diurnal and annual motions of the Earth on its inclined axis, which kept its parallelifm in its whole courſe round the Sun; the motions and phaſes of the Moon, with the retrograde motion of the Nodes of her orbit; and conſequently, all the variety of ſeaſons, the different lengths of days and nights, the days of the New and Full Moons, and Eclipſes.

When it was all completed, except the box that covers the wheels, I ſhewed it to Mr. Maclaurin, who commended it in preſence of a great many young gentlemen who attended his lectures. He deſired me to read them a lecture on it, which I did without any heſitation, ſeeing I had no reaſon to be afraid of ſpeaking before a great and good man who was my friend.—Soon after that I ſent it in a preſent to the Reverend and ingenious Mr. Alexander Irvine, one of the miniſters at Elgin in Scotland.

I then made a ſmaller and neater Orrery, of which all the wheels were of ivory, and I cut the teeth in them with a file.—This was done in the beginning of the year 1743; and, in May that year, I brought it with me to London, where it was ſoon after bought by Sir Dudley Rider. I have made ſix Orreries ſince that time, and there are not any two of them in which the wheel-work is alike: for I could never bear to copy one thing of that kind from another, becauſe I ſtill ſaw there was great room for improvements.

I had a letter of recommendation from Mr. Baron Edlin at Edinburgh to the Right Honourable Stephen Poyntz, Eſq; at St. James's, who had been preceptor to his Royal Highneſs the late Duke of Cumberland, and was well known to be poſſeſſed of all the good qualities that can adorn a human mind.—To me, his goodneſs was really beyond my power of expreſſion; and I had not been a month in London till he informed me that he had wrote to an eminent profeſſor of mathematics to take me into his houſe, and give me board and lodging, with all proper inſtructions to qualify me for teaching a mathematical ſchool he (Mr. Poyntz) had in view for me, and would get me ſettled in it. This I ſhould have liked very well, eſpecially as I began to be tired of drawing pictures, in which, I confeſs, I never ſtrove to excel, becauſe my mind was ſtill purſuing things more agreeable. He ſoon after told me he had juſt received an anſwer from the mathematical maſter, deſiring I might be ſent immediately to him. On hearing this, I told Mr. Poyntz, that I did not know how to maintain my wife during the time I muſt be under the maſter's tuition. What, ſays he, are you a married man? I told him I had been ſo ever ſince May in the year 1739. He ſaid he was ſorry for it, becauſe it quite defeated his ſcheme; as the maſter of the ſchool he had in view for me muſt be a batchelor.

He then aſked me, what buſineſs I intended to follow? I anſwered, that I knew of none beſides that of drawing pictures. On this he deſired me to draw the pictures of his lady and children, that he might ſhew them in order to recommend me to others; and told me, that, when I was out of buſineſs, I ſhould come to him, and he would find me as much as he could: and I ſoon found as much as I could execute: but he died in a few years after, to my inexpreſſible grief.

Soon afterward, it appeared to me, that although the Moon goes round the Earth, and that the Sun is far on the outſide of the Moon's orbit, yet the Moon's motion muſt be in a line that is always concave toward the Sun: and upon making a delineation repreſenting her abſolute path in the Heavens, I found it to be really ſo. I then made a ſimple machine for delineating both her path and the Earth's on a long paper laid on the floor. I carried the machine and delineation to the late Martin Folkes, Eſquire, Preſident of the Royal Society, on a Thurſday afternoon. He expreſſed great ſatisfaction at ſeeing it, as it was a new diſcovery; and took me that evening with him to the Royal Society, where I ſhewed the delineation, and the method of doing it.

When the buſineſs of the Society was over, one of the members deſired me to dine with him next Saturday at Hackney; telling me that his name was Ellicott, and that he was a watchmaker.

I accordingly went to Hackney, and was kindly received by Mr. John Ellicott, who then ſhewed me the very ſame kind of delineation, and part of the machine by which he had done it; telling me that he had thought of it twenty years before. I could eaſily ſee, by the colour of the paper, and of the ink lines upon it, that it muſt have been done many years before I ſaw it. He then told me what was very certain, that he had neither ſtolen the thought from me, nor had I from him. And from that time till his death, Mr. Ellicott was one of my beſt friends. The figure of this machine and delineation is in the 7th Plate of my book of Aſtronomy.

Soon after the ſtile was changed, I had my Rotula new engraved; but have neglected it too much by not fitting it up and advertiſing it. After this, I drew out a ſcheme, and had it engraved, for ſhewing all the problems of the Rotula except the Eclipſes: and, in place of that, it ſhews the times of riſing and ſetting of the Sun, Moon, and Stars; and the poſitions of the Stars for any time of the night.

In the year 1747, I publiſhed a Diſſertation on the Phenomena of the Harveſt Moon, with the deſcription of a new Orrery, in which there are only four wheels. But having never had grammatical education, nor time to ſtudy the rules of juſt compoſition, I acknowledge that I was afraid to put it to the preſs; and, for the ſame cauſe, I ought to have the ſame fears ſtill. But having the pleaſure to find that this my firſt work was not ill received, I was emboldened to go on, in publiſhing my Aſtronomy, Mechanical Lectures, Tables and Tracts relative to ſeveral Arts and Sciences, The Young

Gentleman and Lady's Aſtronomy, a ſmall treatiſe on Electricity, and the following ſheets.

In the year 1748, I ventured to read Lectures on the Eclipſe of the Sun that fell on the 14th of July in that year. Afterwards I began to read Aſtronomical Lectures on an Orrery which I made, and of which the figures of all the wheel-work are contained in the 6th and 7th Plates of this book. I next began to make an apparatus for Lectures on Mechanics, and gradually increaſed the apparatus for other parts of Experimental Philoſophy, buying from others what I could not make for myſelf, till I brought it to its preſent ſtate.—I then entirely left off drawing pictures, and employed myſelf in the much pleaſanter buſineſs of reading Lectures on Mechanics, Hydroſtatics, Hydraulics, Pneumatics, Electricity, and Aſtronomy: in all which, my encouragement has been greater than I could have expected.

The beſt machine I ever contrived is the *Eclipſareon*, of which there is a figure in the 13th Plate of my Aſtronomy. It ſhews the time, quantity, duration, and progreſs of Solar Eclipſes, at all parts of the earth. My next beſt contrivance is the Univerſal Dialing Cylinder, of which there is a figure in the 8th Plate of the Supplement to my Mechanical Lectures.

It is now thirty years ſince I came to London; and during all that time, I have met with the higheſt inſtances of friendſhip from all ranks of people both in town and country, which I do here acknowledge with the utmoſt reſpect and gratitude; and particularly the goodneſs of our preſent gracious Sovereign, who, out of his privy purſe, allows me fifty pounds a-year, which is regularly paid without any deduction.

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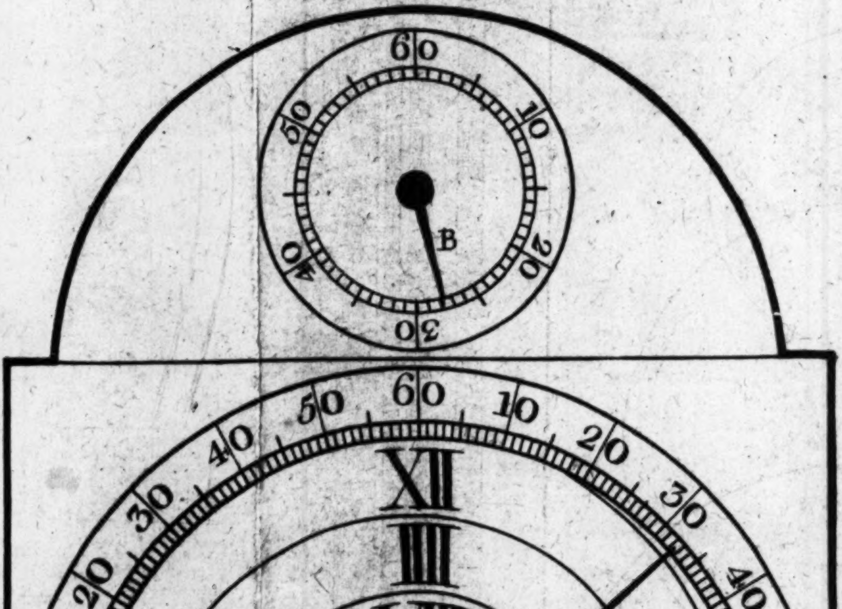
PLATE I.

Fig. 1.



Dr. B. Franklin inv.

Fig. 2.

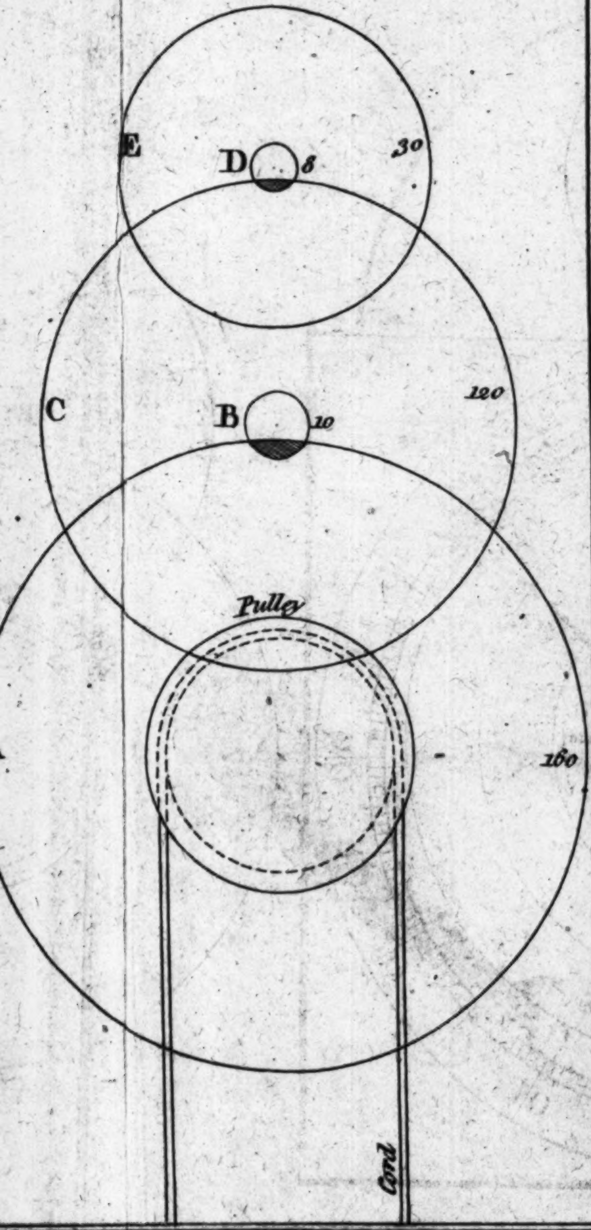
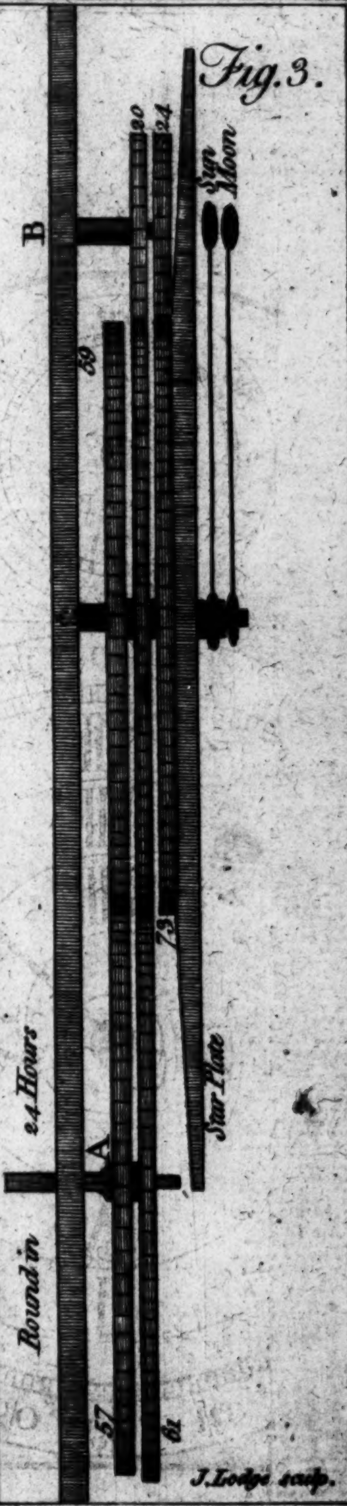


Fig. 3.



J. Lodge sculp.

SELECT EXERCISES.

A Clock ſhewing the Hours, Minutes, and Seconds, having only three Wheels and two Pinions in the whole Movement. Invented by Dr. FRANKLIN *of* Philadelphia.

THE dial-plate of this clock is repreſented by Fig. 1. of Plate I. The hours are engraven in ſpiral ſpaces, along two diameters of a circle containing four times 60 minutes. The index *A* goes round in four hours, and counts the minutes from any hour it has paſſed by, to the next following hour. The time, as it appears in the figure, is either 32½ minutes paſt XII, or paſt IIII, or paſt VIII; and ſo on in each quarter of the circle, pointing to the number of minutes after the hours the index laſt left in its motion. Now, as one can hardly be four hours miſtaken in eſtimating the time, he can always tell the true hour and minute, by looking at the clock, from the time he riſes till the time he goes to bed. The ſmall hand *B*, in the arch at top, goes round once in a minute, and ſhews the ſeconds as in a common clock.

Fig. 2. ſhews the wheel-work of this clock. *A* is the firſt or great wheel, it contains 160 teeth, goes round in four hours, and the index *A* (Fig. 1.) is put upon its axis, and moved round in the ſame time. The hole in the index is round, it is put tight upon the round end of the axis, ſo as to be carried by the motion of the wheel, but may be ſet at any time to the proper hour and minute, without affecting either the wheel or its axis. This wheel of 160 teeth turns a pinion

B of

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B of 10 leaves; and as 10 is but a 16th part of 160, the pinion goes round in a quarter of an hour. On the axis of this pinion is the wheel *C* of 120 teeth; it alſo goes round in a quarter of an hour, and turns a pinion *D*, of 8 leaves, round in a minute; for there are 15 minutes in a quarter of an hour, and 8 times 15 is 120. On the axis of this pinion is the *ſecond-hand* B (Fig. 1.) and alſo the common wheel *E* (Fig. 2.) of 30 teeth, for moving a pendulum (by pallets) that vibrates ſeconds, as in a common clock.

This clock is not deſigned to be wound up by a winch, but to be drawn up like a clock that goes only 30 hours. For this purpoſe, the line muſt go over a pulley on the axis of the great wheel, as in a common 30 hour clock. Several clocks have been made according to this ingenious plan of the Doctor's, and I can affirm, that I have ſeen one of them, which meaſures time exceedingly well.——

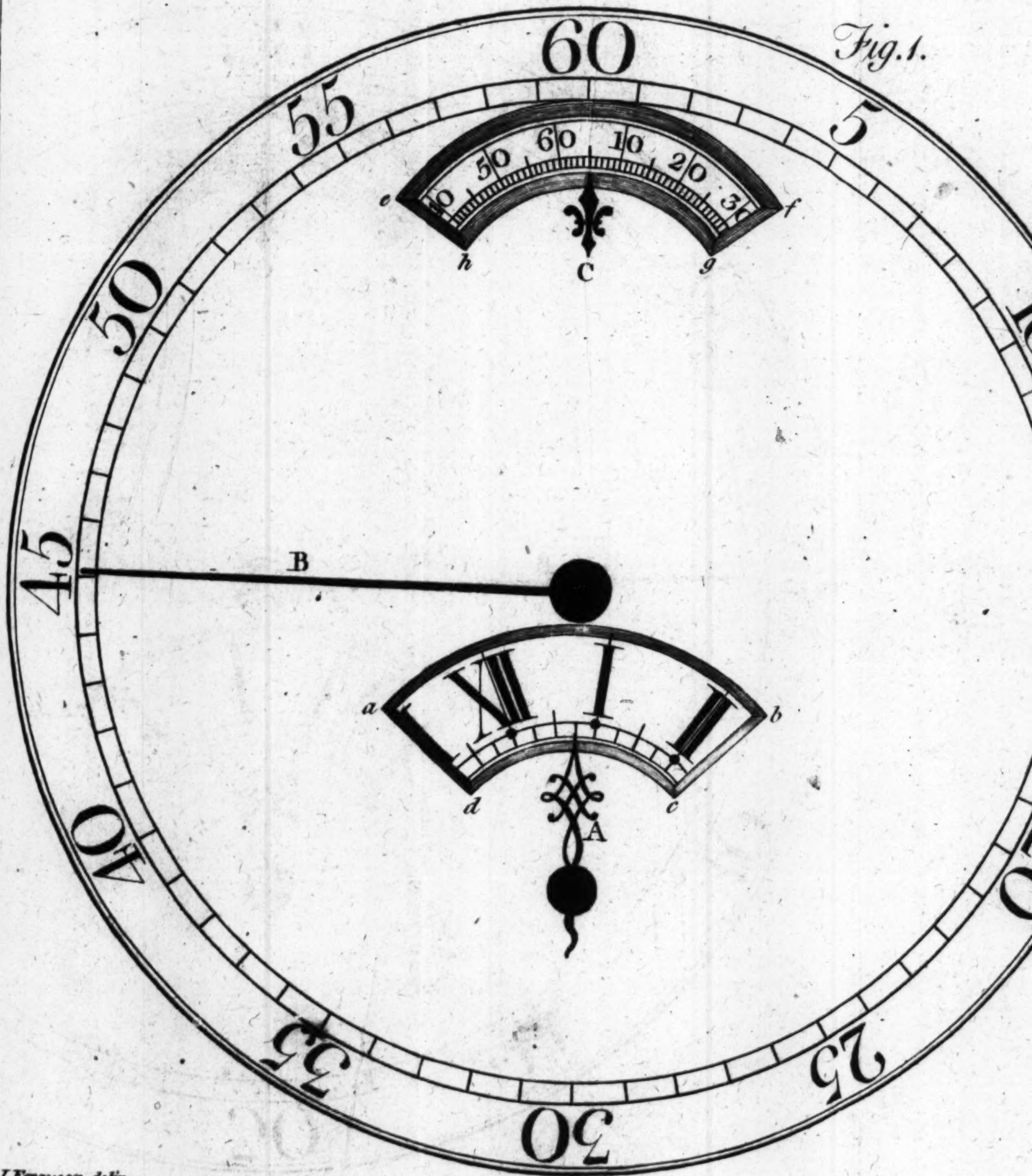
The ſimpler that any machine is, the better it will be allowed to be, by every man of ſcience.

Another Clock that ſhews the Hours, Minutes, and Seconds, by means of only three Wheels and two Pinions in the whole Movement.

As Dr. FRANKLIN, whom I rejoice to call my friend, is perhaps the laſt perſon in the world, who would take any thing amiſs that looks like an amendment or improvement of any ſcheme he propoſes, I have ventured to offer my thoughts concerning his clock, and how one might be made as ſimple as his, with ſome advantages. But I muſt confeſs, that my alteration is attended with ſome inconveniences, of which his are entirely free.—I ſhall mention both, to the beſt of my knowledge, that they who chuſe to have ſuch ſimple and cheap

PLATE II.

Fig. 1.



J. Ferguson delin.

g.1.

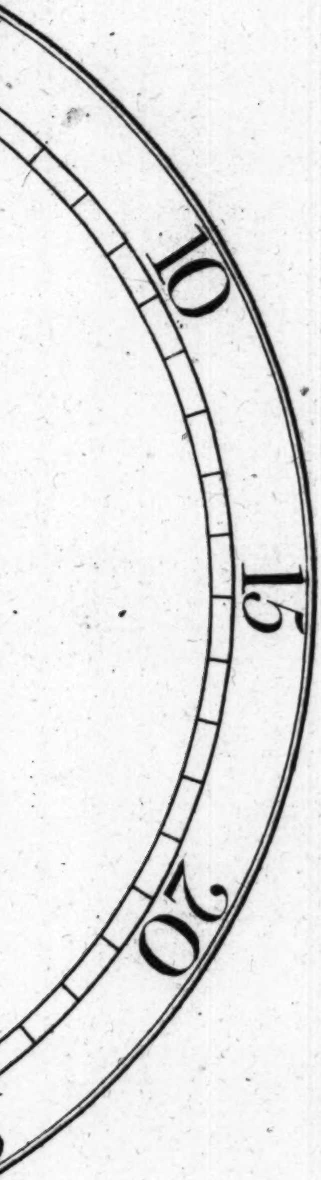
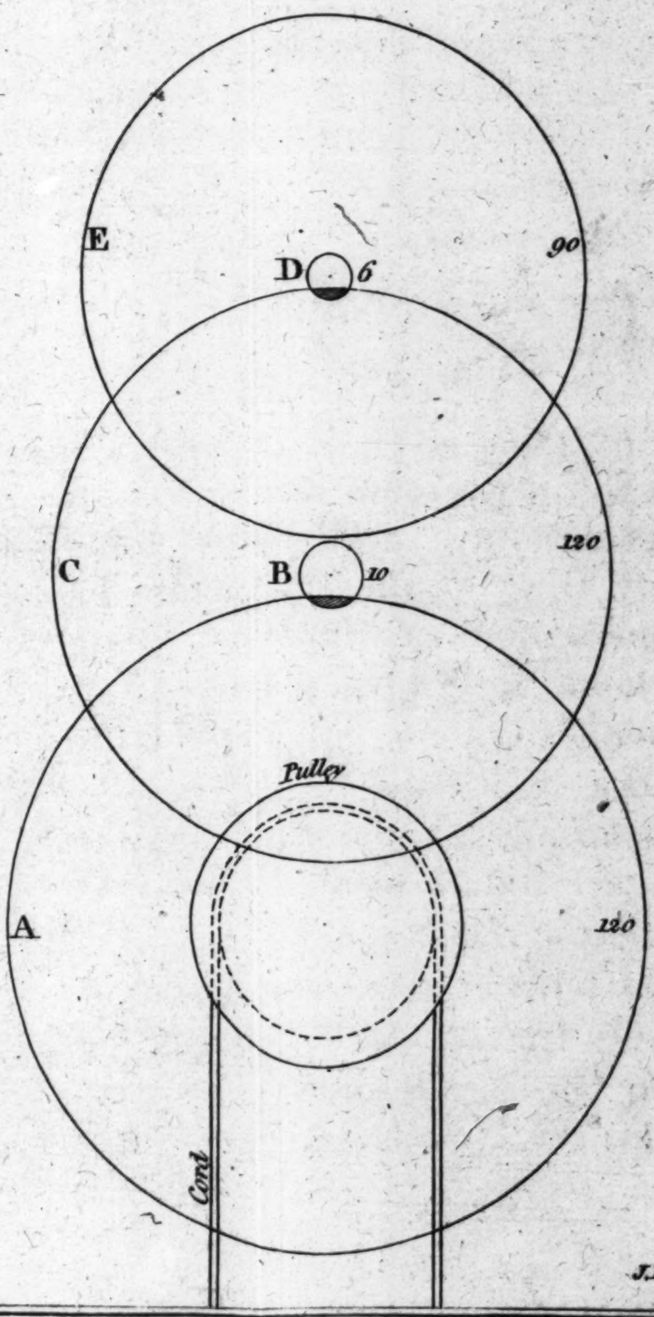


Fig. 2.



J. Lodge sculp.

cheap clocks may have them made in either way they pleaſe.

The Doctor's clock cannot well be made to go a week without drawing up the weight; and if a perſon wakes in the night, and looks at the clock, he may poſſibly be miſtaken four hours in reckoning the time by it, as the hand cannot be upon any hour, or paſs by any hour, without being upon or paſſing by four hours at the ſame time. To avoid theſe inconveniences, I have thought of the following method.

In Fig. 1. of Plate II. the dial-plate of ſuch a clock is repreſented; in which there is an opening *a b c d* below the center. Through this opening, part of a flat plate appears, on which the twelve hours are engraved, and divided into quarters. This plate is contiguous to the back of the dial-plate, and turns round in 12 hours; ſo that the true hour, or part thereof, appears in the middle of the opening, at the point of an index *A*, which is engraved on the face of the dial plate. —*B* is the minute-hand, as in a common clock, going round through all the 60 minutes on the dial in an hour; and, in that time, the plate ſeen thro' the opening *a b c d* ſhifts one hour under the fixt engraven Index *A*. By theſe, you always know the hour and minute, at whatever time you view the dial-plate.—In this Plate is another opening *e f g h*, thro' which the ſeconds are ſeen on a flat moveable ring, almoſt contiguous to the back of the dial-plate; and, as the ring turns round, the ſeconds upon it are ſhewn by the top-point of a *fleur-de-lis* C, engraved on the face of the dial-plate.

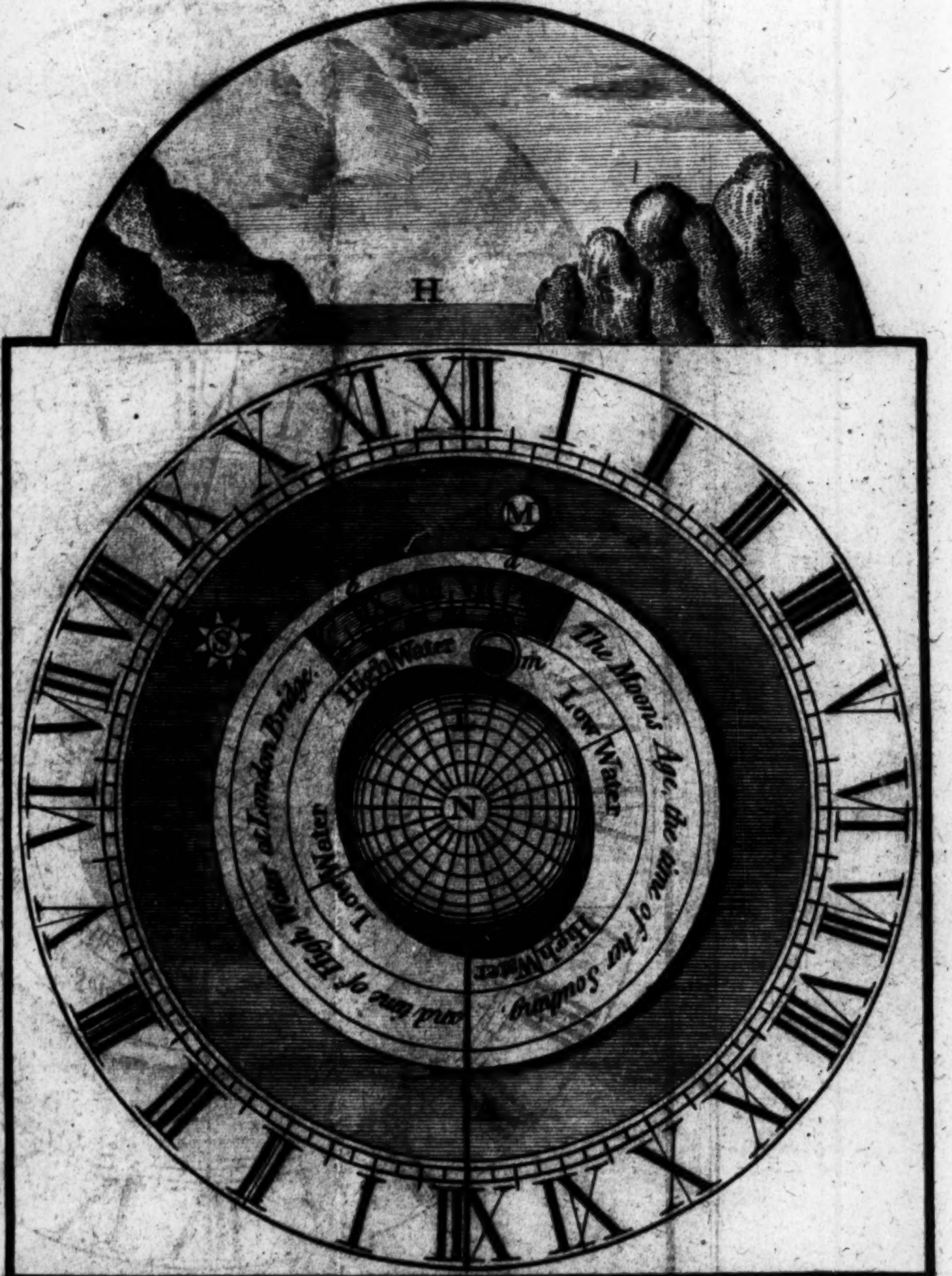
Fig. 2. repreſents the wheels and pinions in this clock. *A* is the firſt or great wheel; it contains 120 teeth, and turns round in 12 hours. On its axis is the plate on which the 12 hours above mentioned are engraved. This plate is not fixed on the axis, but is only put tight upon a round part thereof, ſo that any hour, or part of an hour, may be ſet to the top of the fixed index *A* (Plate I.) without affecting the motion of the wheel. For this purpoſe, twelve ſmall holes are drilled through the plate, one at each hour, among the quarter diviſions: and, by putting a pin into any hole in view, the plate may be ſet, without affecting any part of the wheel-work. This great wheel *A*, of 120 teeth, turns a pinion *B* of 10 leaves round in an hour; and the minute-hand *B* (Fig. 1.) is on the axis of this pinion, the end of the axis not being ſquare, but round; that the minute-hand may be turned occaſionally upon it, without affecting any part of the movement. On the axis of the pinion *B* is a wheel *C* of 120 teeth, turning round in an hour, and turning a pinion *D* of 6 leaves in 3 minutes; for 3 minutes is a 20th part of an hour, and 6 is a 20th part of 120. On the axis of this pinion is a wheel *E* of 90 teeth, going round in 3 minutes, and keeping a pendulum in motion that vibrates ſeconds, by pallats, as in a common clock, where the pendulum wheel has only 30 teeth, and goes round in a minute.—But, as this wheel goes round only in 3 minutes, if we want it to ſhew the ſeconds, a thin plate muſt be divided into 3 times 60, or 180 equal parts, and numbered 10, 20, 30, 40, 50, 60; 10 20, 30, 40, 50, 60; 10, 20, 30, 40, 50, 60; and fixed upon the ſame axis with the wheel of 90 teeth, ſo near the back of the dial-plate, as only to turn round without touching it: and theſe diviſions will ſhew the ſeconds, thro' the opening *e f g h* in the dial-plate, as they ſlide gradually round below the point of the fixed *fleur-de-lis* C.

As the great wheel *A*, and pulley on its axis over which the cord goes (as in a common 30 hour clock) turns round only once in 12 hours, this clock will go a week with a cord of common length, and always have the true hour, or part of that hour, in ſight at the upper end of the fixed index *A* on the dial-plate. Theſe are two advantages it has beyond Dr. *Franklin*'s clock: but it has two diſadvantages of which his clock is free. For, in this, although the 12 hour wheel turns the minute-index *B*, yet, if *that* index be turned by hand, to ſet it to the proper minute for any time, it will not move the 12 hour plate to ſet the correſponding part of the hour even with the top of the index *A*: and therefore, after having ſet the minute-index *B* right by hand, the hour-plate muſt be ſet right by means of a pin put into the ſmall hole in the plate juſt below the hour. 'Tis true, there is no great matter in this; but I have ſome ſuſpicion that the pendulum wheel *E* having 90 teeth inſtead of the common number 30, may be ſome diſadvantage to the ſcapement, on account of the ſmallneſs of the teeth; and 'tis certain, that it will cauſe the pendulum-ball to deſcribe but ſmall arcs in its vibrations. Indeed ſome men of ſcience think ſmall arcs are beſt; but if they really are, I muſt confeſs myſelf ignorant of the reaſon. For, whether the ball deſcribes a large or a ſmall arc, if the arc be nearly *cycloidal*, the vibrations will be performed in equal times; the time then depending entirely on the length of the pendulum-rod, not on the length of the arc the ball deſcribes. The larger the arc is, the greater is the *momentum* of the ball; and the greater the *momentum* is, the leſs will the times of the vibrations be affected by any unequal impulſe of the pendulum-wheel upon the pallets.

But the worſt thing about this clock (and what every one will allow to be a diſadvantage) is, that the weight of the flat ring on which the ſeconds are engraved,

PLATE III.

Fig. 1.


H
M
S
N
m
e
d
XII
I
II
III
IIII
V
VI
VII
VIII
IX
X
XI
The Moons Age, the time of her Southing, and time of High Water at London Bridge
High Water
Low Water
High Water
Low Water

J. Ferguson inv. et delin.

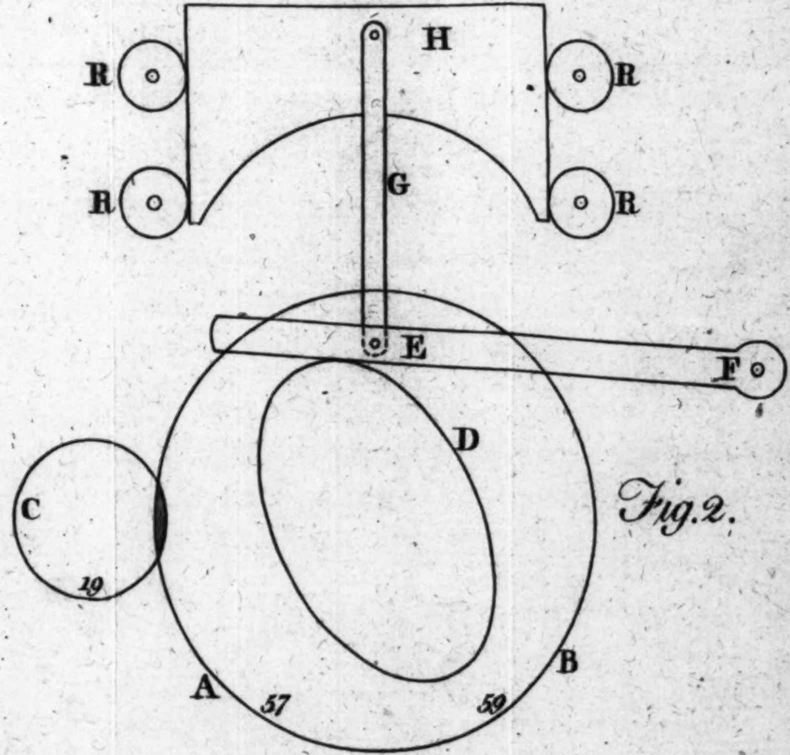


Fig. 2.

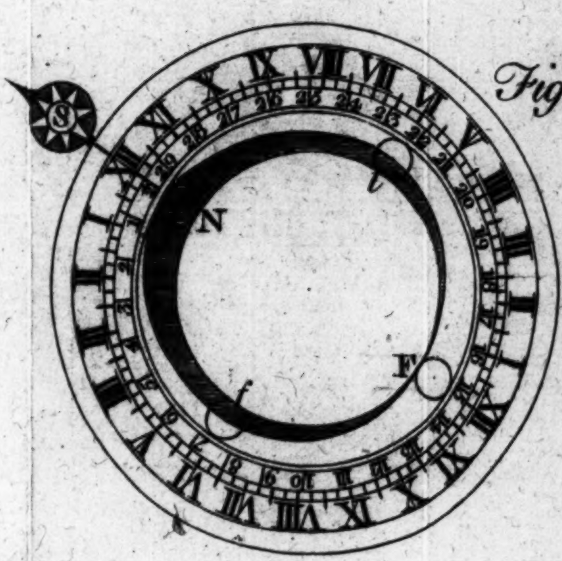


Fig. 3.

J. Lodge sc.

engraved, will load the pivots of the axis of the pendulum-wheel with a great deal of friction, which ought by all poſſible means to be avoided; and yet I have ſeen one of theſe clocks (lately made) that goes very well, notwithſtanding the weight of this ring. For my own part, I think it might be quite left out; for ſeconds are of very little uſe in common clocks not made for aſtronomical obſervations; and table-clocks never have them.

A Clock ſhewing the apparent daily Motions of the Sun and Moon, the Age and Phaſes of the Moon, with the Time of her coming to the Meridian, and the Times of High and Low Water, by having only two Wheels and a Pinion added to the common Movement.

The dial-plate of this clock is repreſented by Fig 1. of Plate III. It contains all the 24 hours of the day and night. *S* is the Sun, which ſerves as an hour-index, by going round the dial-plate in 24 hours; and *M* is the Moon, which goes round in 24 hours 50½ minutes, from any point in the hour-circle to the ſame point again, which is equal to the time of the Moon's going round in the heavens, from the meridian of any place to the ſame meridian again. The Sun is fixed to a circular plate (as Fig. 3.) and carried round by the motion of that plate, on which the 24 hours are engraven, and within them is a circle divided into 29½ equal parts for the days of the Moon's age, accounted from the time of any new Moon to the next after; and each day ſtands directly under the time (in the 24 hour circle) of the Moon's coming to the meridian, the XII under the Sun ſtanding for mid-day, and the oppoſite XII for midnight.—Thus, when the Moon is 8 days old, ſhe comes to the meridian at half an hour paſt VI in the afternoon; and when ſhe is 16 days old, ſhe comes to the meridian at I o'clock in the morning. The Moon *M* (Fig. 1.) is fixed to another circular plate, of the ſame diameter with that which carries the Sun; and this Moon-plate turns round in 24 hours $50\frac{1}{2}$ minutes. It is cut open, ſo as to ſhew ſome of the hours, and days of the Moon's age, on the plate below it that carries the Sun, and, acroſs this opening, at *a* and *b* are two ſhort pieces of ſmall wire in the Moon-plate. The wire *a* ſhews the day of the Moon's age, and time of her coming to the meridian, on the plate below it that carries the Sun; and the wire *b* ſhews the time of high water for that day, on the ſame plate. Theſe wires muſt be placed as far from one another, as the time of the Moon's coming to the meridian differs from the time of high water at the place where the clock is intended to ſerve. At London-bridge, it is high-water when the Moon is two hours and an half paſt the meridian.

Above this plate, that carries the Moon, there is a fixed plate *N*, ſupported by a wire *A*, the upper end of which is fixed to that plate, and the lower end is bent to a right angle, and fixed into the dial-plate at the lowermoſt or midnight XII. This plate may repreſent the Earth, and the dot at *L* London, or any other place at which the clock is deſigned to ſhew the times of high and low water.

Around this plate is an elliptical ſhade upon the plate that carries the Moon *M*: the higheſt points of this ſhade are marked *High Water*, and the loweſt points *Low Water*. As this plate turns round below the fixed plate *N*, the high and low water points come ſucceſſively even with *L*, and ſtand juſt over it at the times when it is high or low water at the given place; which times are pointed out by the Sun *S*, among the 24 hours on the dial-plate:

and, in the arch of this plate, above XII at noon, is a plate *H* that riſes and falls as the tide does at the given place. Thus, when it is high water (ſuppoſe at London) one of the higheſt points of the elliptical ſhade ſtands juſt over *L*, and the tide-plate *H* is at its greateſt height: and when it is low water at London, one of the loweſt points of the elliptical ſhade ſtands over *L*, and the tide-plate *H* is quite down, ſo as to diſappear beyond the dial-plate.

As the Sun *S* goes round the dial-plate in 24 hours, and the Moon *M* goes round it in 24 hours 50½ minutes, the Moon goes ſo much ſlower than the Sun as to make only 28½ revolutions in the time the Sun makes 29½; and therefore the Moon's diſtance from the Sun is continually changing; ſo that, at whatever time the Sun and Moon are togeher, or in conjunction, in 29½ days afterward they will be in conjunction again. Conſequently, the plate that carries the Moon moves ſo much ſlower than the plate that carries the Sun, as always to make the wire *a* ſhift over one day of the Moon's age on the Sun's plate in 24 hours.

In the plate that carries the Moon, there is a round hole *m*, thro' which the phaſe or appearance of the Moon is ſeen on the Sun's plate, for every day of the Moon's age from change to change. When the Sun and Moon are in conjunction, the whole ſpace ſeen through the hole *m* is black: when the Moon is oppoſite to the Sun (or full) all that ſpace is white: when ſhe is in either of her quarters, the ſame ſpace is half black half white: and different in all other poſitions, ſo as the white part may reſemble the viſible or enlightened part of the Moon for every day of her age.

To ſhew theſe various appearances of the Moon, there is a black ſhaded ſpace (Fig. 3.) as *N f F l*, on the plate that carries the Sun. When the Sun and Moon are in conjunction, the whole ſpace ſeen through the round hole is black, as at *N:* when the Moon is full, oppoſite to the Sun, all the ſpace ſeen through the round hole is white, as at *F:* when the Moon is in her firſt quarter, as at *f*, or in her laſt quarter, as at *l*, the hole is only half ſhaded; and more or leſs accordingly for each poſition of the Moon with regard to her age; as is abundantly plain by the Figure.

The wheel-work and tide-work of this clock is repreſented by Fig. 2. in which *A* and *B* are two wheels of equal diameters. *A* has 57 teeth, its axis is hollow, it comes through the dial of the clock, and carries the Sun-plate with the Sun (*S*, in Fig. 1.). *B* has 59 teeth, its axis is a ſolid ſpindle, turning within the hollow axis of *A*, and carrying the Moon-plate with the Moon (*M*, in Fig. 1.). A pinion *C* of 19 leaves takes into the teeth of both the wheels, and turns them round. This pinion is turned round, by the common clock-work, in 8 hours; and, as 8 is a third part of 24, ſo 19 is a third part of 57: and therefore the wheel *A*, of 57 teeth, that carries the Sun, will go round in 24 hours exactly. But, as the ſame pinion *C* (that turns the wheel *A* of 57 teeth) turns alſo the wheel *B* of 59 teeth, this laſt wheel will not turn round in leſs than 24 hours 50 $\frac{1}{2}$ minutes of time: for as 57 teeth are to 24 hours, ſo are 59 teeth to 24 hours 50$\frac{1}{2}$ minutes, very nearly.

On the back of the Moon-wheel of 59 teeth is fixed an elliptical ring *D*, which, as it turns round, raiſes and lets down a lever *E F*, whoſe center of motion is on a pin at *F*; and this, by means of an upright bar *G*, raiſes and lets down the tide-plate *H*, twice in the time of the Moon's revolving from the meridian to the meridian again. The upper edge of this plate is ſhewn at *H*, in Fig. 1 and it moves between four rollers *R*, *R*, *R*, *R*, in Fig. 2.

I have made one of theſe clocks to go by the movement of an old watch, in the following manner:

The firſt, or great wheel of a watch, goes round in four hours. I put a wheel of 20 teeth on the end of the axis of that wheel, to turn a wheel of 40 teeth on the axis of the pinion *C*; by which means, *that* pinion is turned round in 8 hours, the wheel *A* in 24 hours, and the wheel *B* in 24 hours 50½ minutes.—I never ſaw nor heard of any other clock of this kind.

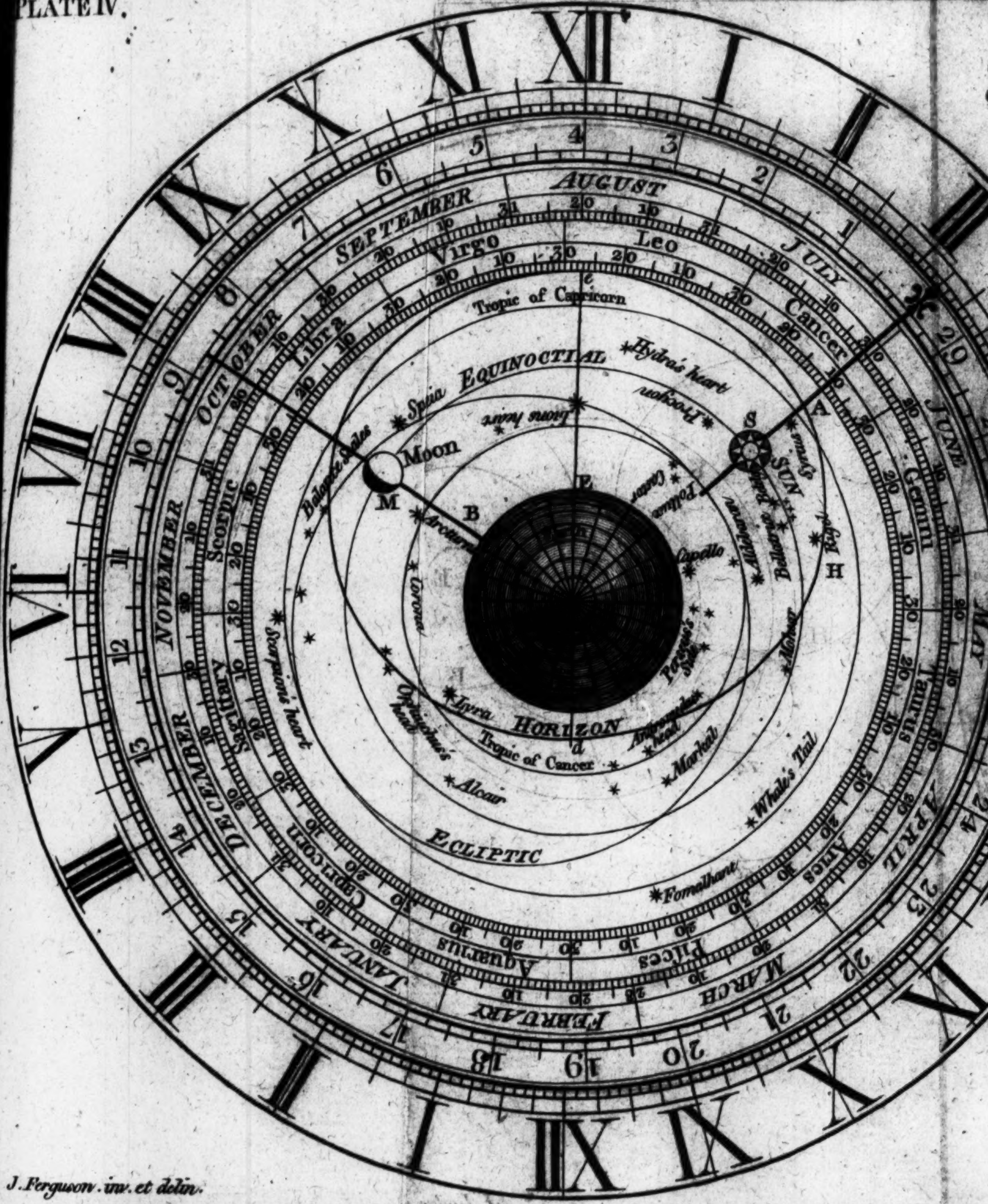
An aſtronomical Clock, ſhewing the apparent daily Motions of the Sun, Moon, and Stars with the Times of their riſing, ſouthing, and ſetting, the Places of the Sun and Moon in the Ecliptic, and the Age and Phaſes of the Moon, for every Day of the Year.

The dial-plate of this clock is repreſented by Fig. 1. of Plate IV. It contains all the 24 hours of the day and night, and each hour is divided into 12 equal parts, ſo that each part anſwers to 5 minutes of time.

Within theſe diviſions of the hour-circle is a flat ring, the face of which is juſt even (or in the ſame plane) with the face of the hour-circle. This ring is divided into 29½ equal parts (numbered 1, 2, 3, 4, &c. from the right hand toward the left) which are the days of the Moon's age from change to change: the ring turns round in 24 hours, and has a *fleur-de-lis* upon it, ſerving as an hour-index to point out the time of the day or night in the 24 hour circle.

Within this ring, and about four-tenths of an inch below its flat ſurface, is a flat circular plate, on which the months and days of the year are engraved; and within theſe, on the ſame plate, is a circle containing the ſigns and degrees of the ecliptic, divided in ſuch a manner, as that each

PLATE IV.



J. Ferguson inv. et delin.

Fig. 1.

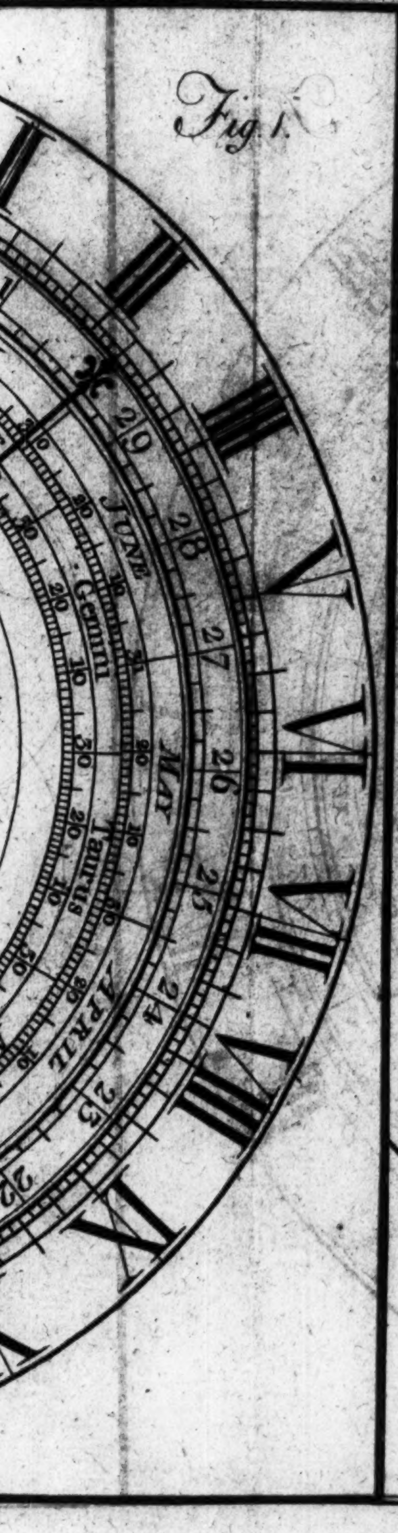
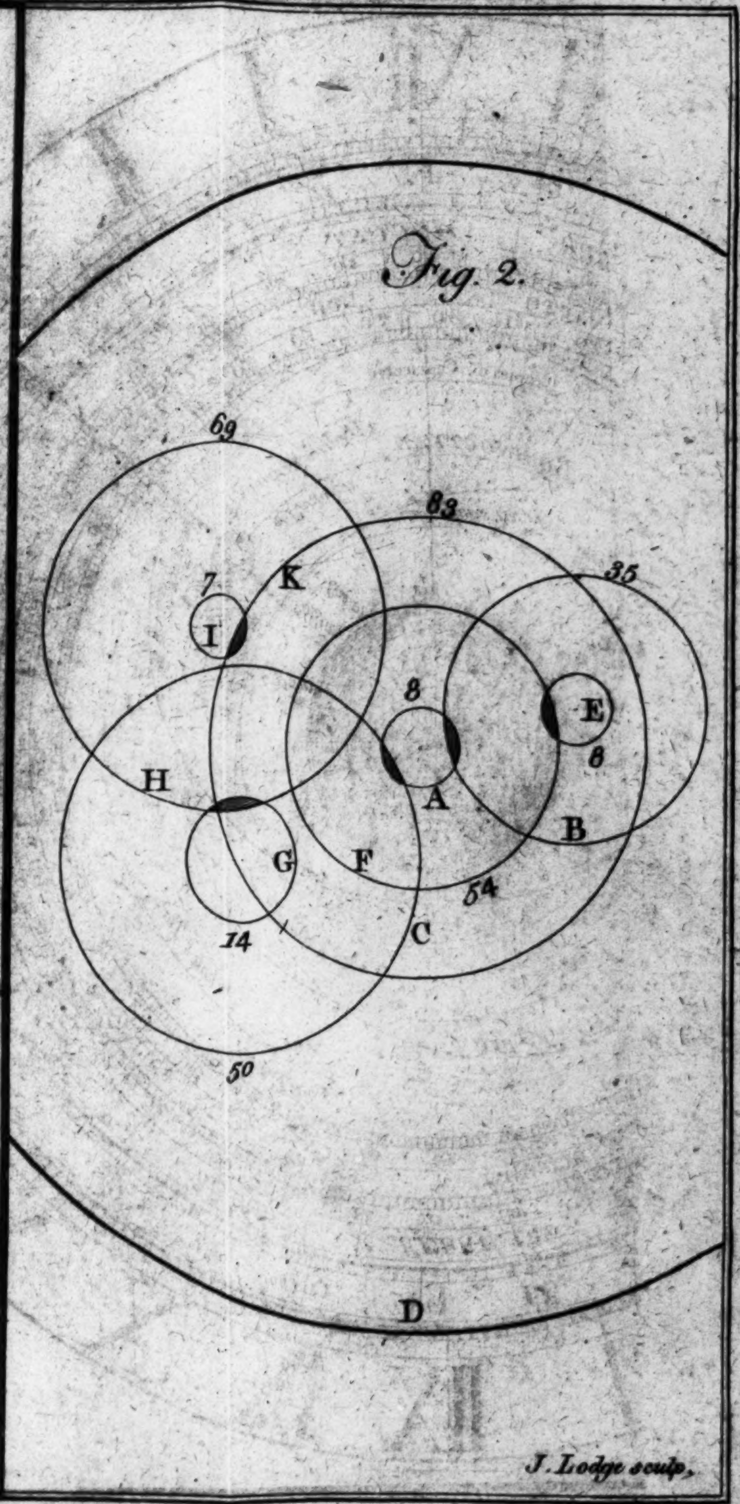


Fig. 2.



J. Lodge sculp.

particular day of the year ſtands over the ſign and degree of the Sun's place on that day.

Within this circle, on the ſame plate, the ecliptic, equinoctial, and tropics, are laid down; and all the ſtars of the firſt, ſecond, and third magnitude that are ever ſeen in the latitude of London, according to their reſpective right aſcenſions and declinations; thoſe of the firſt magnitude being diſtinguiſhed by eight points, thoſe of the ſecond by ſix, and thoſe of the third by five. This plate turns round in 23 hours 56 minutes 4 ſeconds 6 thirds of time, which is the length of a ſydereal day; and conſequently it makes 366 revolutions (as the ſtars do in the heavens) in the time the Sun makes 365; the number of ſydereal days in a year exceeding the number of ſolar days by one.

Over the middle of this plate, and about four tenths of an inch from it, is a fixed plate *E*, to repreſent the

Earth; round which, the Sun, Moon, and Stars move in their proper times, *viz.* the Sun in 24 hours, the Moon in 24 hours 50½ minutes, and the Stars in 23 hours 56 minutes 4 ſeconds 6 thirds. The Sun *S* is carried round by a wire *A*, which is fixed into the inſide of the Moon's age ring, even with the *fleur-de-lis*; the Moon *M* is carried round by a wire *B*, which is fixed to the axis of a wheel below the Earth *E*, and the Star-plate is turned round by a wheel at the back of the dial-plate.

Over the dial-plate is a glaſs, as in common clocks. On this glaſs is an ellipſis *H*, drawn with a diamond, to repreſent the horizon of the place for which the clock is to ſerve, and acroſs this horizon is a ſtraight line *e E d* (even with the two XII's) to repreſent the meridian. All the Stars that are ſeen at any time within this ellipſis are above the horizon at that time, and all thoſe that are without it are then below the horizon. When any Star on the Plate comes to the left-hand ſide of the horizon (the Stars moving from left to right) the like Star in the heavens is riſing; when it comes under the meridian line *e E*, the like Star in the heavens is on the meridian of the place; and when it comes to the right-hand ſide of the horizon, the like Star in the heavens is ſetting.

When the point of the ecliptic, that the Sun's wire *A* interſects, comes to the left-hand ſide of the horizon, on any day of the year cut by that wire, the *fleur-de-lis* will be at the time of the Sun's riſing, in the 24 hour circle; and at the time of his ſetting when the interſection of the Sun's wire and ecliptic comes to the weſtern ſide of the horizon.—The like is to be underſtood with regard to the riſing and ſetting of the Moon, when the point of the ecliptic which her wire *B* interſects

ſects comes to the left and right hand ſides of the horizon.

Every 24 hours, the Moon's wire ſhifts over one day of her age in the circle of $29\frac{1}{2}$ equal parts on the flat ring above mentioned. Each of theſe day-ſpaces is divided into four equal parts, for ſhewing the Moon's age to every ſixth hour thereof. Thus, as the Moon's wire *B* ſtands in the figure, it ſhews the Moon to be 8 days 18 hours old. It ſhifts quite round the ring, and carries the Moon round from the Sun to the Sun again, in 29 days 12 hours and 45 minutes.

The Sun, on its wire *A*, goes 365 times round in 365 days; and, in that time, the Star-plate, with the months and days of the year upon it, goes 366 times round. So that, for every revolution of the Sun, the Star-plate advances forward, under the Sun's wire, through the ſpace of one day in the circle of months: and by this means the Sun's wire ſhews the day of the month throughout the whole year; and at the ſame time, for each particular day of the year, it ſhews the Sun's place in the ecliptic, in the circle of ſigns.—The Moon's wire *B* cuts the Moon's place in the circle of ſigns, for every day of her age, throughout the year.

The whole circle of ſigns ſhifts round, under the Sun's wire *A*, in 365 days 5 hours 48 minutes 58 ſeconds, which is the time the Sun takes in going quite round the ecliptic: and the Moon's wire ſhifts over all the circle of ſigns in 27 days 7 hours and 43 minutes, which is the time of the Moon's going round the ecliptic. And thus, by theſe different motions of the Sun and Moon, there are always 29 days 12 hours and 45 minutes between any conjunction of the Sun and Moon, and the next ſucceeding one.

The moon *M* is a round ball, half black half white: it turns round its axis, or wire *B*, in 29 days 12 hours 45 minutes, and ſo ſhews all the different phaſes of the Moon, for every day of her age, from change to change. When the Sun and Moon are together, or in conjunction, the white ſide of the Moon-ball *M* is toward the Sun, and the black ſide toward the eye of a ſpectator looking at the clock, who then can ſee no part of the white or (apparently enlightened) ſide of the Moon: when the Moon is full, or oppoſite to the Sun, all the white ſide of the ball *M* is turned toward the ſpectator's eye: and when the Moon is in either of her quarters, or 90 degrees from the Sun, the ſpectator ſees half the black and half the white ſide of the ball; ſo that the white part of it then appears like the Moon in her firſt or laſt quarter.

Fig. 2. repreſents (what is called) the dial-work of this clock, or the wheels at the back of the dial-plate, between it and the wheels of the common movement, which are contained between two fixed plates, wherein the pivots of their axis turn in holes.

A long pin or ſpindle is fixed into the movement-plate next the back of the dial-plate, perpendicular to both theſe plates. This ſpindle goes thro' the center of the dial-plate, and has the Earth-plate *E* (Fig. 1.) fixed on the end of it.

On this ſpindle, and cloſe to the movement-plate in which it is fixed, is a fixed pinion *A* of 8 leaves (Fig. 2.) which take into the teeth of a wheel *B*, and of a wheel *C*: the number of teeth in *B* is 35, and the number in *C* is 50.

Theſe two wheels hang and turn upon a large plate *D*; which, by the common clock movement, is turned round upon the axis of the fixed pinion *A* in 24 hours: and conſequently, as theſe wheels are carried round the fixed pinion, and its leaves take into their teeth, each of them will be ſo far turned round its axis, in 24 hours, as is equal to 8 of their teeth. The axis of the plate *D* is hollow, and turns upon the ſolid fixed ſpindle or axis of the pinion *A*; and the ſide of the plate *D* that is neareſt to the clock-movement is almoſt contiguous to the end of the pinion *A* next to the dial-plate of the clock. Hence it is plain, that the two wheels *B* and *C*, which are carried round the pinion *A*, and are turned by it, muſt be on that ſide of the plate *D* which is almoſt contiguous to the pinion. All the reſt of the wheels and pinions in the figure are on the other ſide of the plate *D*; namely, on the ſide of it which is next to the back of the dial-plate.

The axes of the wheels *B* and *C* go through the plate *D*: on the top of the axis *B* is a pinion *E* of 8 leaves, which turns a wheel *F* of 54 teeth. The axis of this wheel is hollow, and turns upon the hollow axis of the plate *D*; it comes through the center of the dial-plate of the clock, and carries the Moon round by the wire *B* in Fig. 1. which wire turns round in a piece that goes tight upon the hollow end of the axis of *F* (Fig. 2.) juſt under the Earth-plate *E* (Fig. 1.).

The hollow axis of the plate *D* turns round, as the plate does, in 24 hours. On the end of this axis, juſt under the middle of the Earth-plate *E*, is a ſmall wheel of 20 teeth, turning a contrate wheel of the ſame number, the pivots of whoſe axis turn in the piece that carries the Moon's wire; and this wheel turns another wheel of the ſame number of teeth, fixed on the Moon's wire or axis. By theſe wheels (which lie concealed under the Earth plate *E*) the Moon is turned round her axis in 29 days 12 hours 45 minutes, as above mentioned, and ſhews her different phaſes. Theſe three laſt mentioned wheels are not repreſented in Fig. 2. becauſe they could not be put in without confuſing it.

On the axis of the wheel *C* of 50 teeth is a pinion *G* of 14 leaves, which turns a wheel *H* of 69 teeth, on whoſe axis is a pinion *I* of 7 leaves, turning a wheel *K* of 83 teeth: this wheel is pinned faſt to the back of the Star-plate, which, together with the wheel, turns round in a ſydereal day, or in 23 hours 56 minutes 4 ſeconds 6 thirds of mean ſolar time; and the above-mentioned wheels, which belong to the Moon, will carry her round from the meridian to the meridian again in 24 hours 50½ minutes; if the plate *D*, on which all the wheels hang, be turned round in 24 mean ſolar hours.

The plate *D* carries the ring of 29½ equal parts round, within the fixed 24 hour-circle of the dial-plate, in 24 hours, by means of four pillars, whoſe oppoſite ends are fixed into the plate *D*, and into the ring. By this, the Sun is carried round in 24 hours (its wire *A* being fixed into the ring) and alſo the *fleur-de-lis* that ſhews the time, like an index, in the 24 hour-circle.

I contrived this clock above twenty years ago, and made a model of it in wood, which I have ſtill in my cuſtody; and ſince that time,

Another Clock ſhewing the Equation of Time, the apparent daily Motions of the Sun, Moon, and Stars; with the Times of their riſing, ſouthing, and ſetting.

The dial-plate of this clock is like the one laſt deſcribed; but the wheel-work at the back of the dial-plate is different, and rather more ſimple.

Two wheels, one of 57 teeth and the other of 61, are fixed upon an axis *A* (ſee Fig. 3. of Plate I.) and turned round by the common clock-movement in 24 hours.

The wheel of 57 teeth turns a wheel of 59, on the top of whoſe axis, above (or before the face of) the Star-plate, is a wire that carries the Moon round, from the meridian to the meridian again in 24 hours $50\frac{1}{2}$ minutes.

The wheel of 61 teeth turns another of the ſame number, whoſe axis is hollow, and turns on the ſolid axis of the wheel of 59. On the top of this hollow axis, between the Moon's wire and the Star-plate, is a wire that carries the Sun round from the meridian to the meridian again, in 24 mean ſolar hours.

The laſt mentioned wheel of 61 teeth (whoſe axis carries the Sun) turns a wheel of 20 teeth, upon a pin fixed in the immoveable plate *B*; and to this wheel of 20 teeth is fixed a wheel of 24, by pins put through them, and riveted on their outſides: this wheel of 24 teeth turns a wheel of 73, looſe on the hollow axis of the Sun-wheel of 61, and riveted faſt to the Star-plate;

which, together with the ſaid wheel, turns 366 times round in 365 days, meaſured by the Sun's motion. The days of the months, and correſponding Equations of Time are laid down on the Star-plate, and cut by the wire that carries the Sun.

How to regulate a Clock by the Motion of the Stars, ſo as to meaſure mean ſolar Time exactly.

As the Stars make 366 revolutions in 365 days, each Star comes to the meridian 3 minutes $55\frac{9}{10}$ ſeconds on each ſucceeding day or night ſooner than it did on the day or night before. We ſhall firſt give the following table of their revolutions for a year, and then ſhew its uſe in regulating clocks and watches, ſo as to keep mean ſolar time.

A TABLE *for regulating Clocks and Watches to true equal Time, by the daily Revolutions of the Stars.*

Rev.	Days.	H.	M.	S.	H.	M.	S.
1	=0	23	56	4.1	0	3	55.9
2	1	23	52	8.2	0	7	51.8
3	2	23	48	12.3	0	11	47.7
4	3	23	44	16.4	0	15	43.6
5	4	23	40	20.5	0	19	39.5
6	5	23	36	24.6	0	23	35.4
7	6	23	32	28.7	0	27	31.3
8	7	23	28	32.8	0	31	27.2
9	8	23	24	36.9	0	35	23.1
10	9	23	20	41.0	0	39	19.0
11	10	23	16	45.1	0	43	14.9
12	11	23	12	49.2	0	47	10.8
13	12	23	8	53.3	0	51	6.7
14	13	23	4	57.4	0	55	2.6
15	14	23	1	1.5	0	58	58.5
16	15	22	57	5.6	1	2	54.4
17	16	22	53	9.7	1	6	50.3
18	17	22	49	13.8	1	10	46.2
19	18	22	45	17.9	1	14	42.1
20	19	22	41	22.0	1	18	38.0
21	20	22	37	26.1	1	22	33.9
22	21	22	33	30.2	1	26	29.8
23	22	22	29	34.3	1	30	25.7
24	23	22	25	38.4	1	34	21.6
25	24	22	21	42.5	1	38	17.5
26	25	22	17	46.6	1	42	13.4
27	26	22	13	50.7	1	46	9.3
28	27	22	9	54.8	1	50	5.2
29	28	22	5	58.9	1	54	1.1
30	29	22	2	3.0	1	57	57.0
40	39	21	22	44.0	2	37	16.0
50	49	20	43	25.0	3	16	35.0
60	59	20	4	6.0	3	55	54.0
70	69	19	24	47.0	4	35	13.0
80	79	18	45	28.0	5	14	32.0
90	89	18	6	9.0	5	53	51.0
100	99	17	26	50.0	6	33	10.0
200	199	10	53	40.0	13	6	20.0
300	299	4	20	30.0	19	39	30.0
360	359	0	24	36.0	23	35	24.0
365	364	0	4	56.5	23	55	3.5
366	365	0	1	0.6	23	58	59.4

The firſt column denotes the number of revolutions of the Stars, from the meridian to the meridian again, in a common year of 365 days: the next other columns (titled *Days H. M. S.)* ſhew the times in which theſe revolutions are made: and thoſe in the right-hand part of the Table (titled *H. M. S.)* ſhew how much any Star gains daily upon the time ſhewn by a well-regulated clock or watch.

Therefore to know whether the clock goes true or not, obſerve the time by the clock when any Star diſappears behind a chimney (or any other fixed object) as ſeen through a hole in a thin plate of metal fixed in a window-ſhutter; and if the ſame Star diſappears on every ſucceeding night as much ſooner by the clock as to agree with the times ſhewn in the right-hand part of the Table (as ſuppoſe 39 minutes 19 ſeconds in 10 days, or 1 hour 18 minutes 38 ſeconds in 20 days)

days) the clock goes true: otherwiſe it does not, and muſt be regulated accordingly, by ſcrewing up or letting down the ball of the pendulum, as it goes too ſlow or too faſt.

To find the length of a Pendulum that ſhall make any given number of Vibrations in a Minute, and vice verſa.

A pendulum whoſe length is 39.2 inches, from the point of ſuſpenſion to the center of oſcillation, makes 60 vibrations in a minute; and this is called the ſtandard length. Then, for any other number of vibrations in a minute, ſay, As the ſquare of the given number of vibrations is to the ſquare of 60, ſo is the length of the ſtandard to the length ſought.—Thus, ſuppoſe the given number of vibrations to be 30 *per* minute: the ſquare of 30 is 900, and the ſquare of 60 is 3600: then, as 900 is to 3600, ſo is 39.2 to 156.8; ſo that the length required for 30 vibrations *per* minute is 156.8 inches.

If the length of the pendulum be given, and the number of vibrations it makes in a minute be required; ſay, As the given length is to the ſtandard length (39.2 inches) ſo is the ſquare of 60 vibrations to the ſquare of the number required: the ſquare root of which ſhall be the number of vibrations made by the pendulum in a minute. Thus, ſuppoſe the given length to be 156.8 inches: As 156.8 is to 39.2, ſo is 3600 (the ſquare of 60) to 900; the ſquare root of which is 30, the number of vibrations that this pendulum will make in a minute.

The length of a pendulum that would make only one vibration in a minute is 3920 yards, or 141120 inches: and the length of a pendulum that would make 240 vibrations in a minute (or 4 in a ſecond) is 2.45 inches.

In theſe calculations it is ſuppoſed that the weight of the pendulum-rod bears little or no ſenſible proportion to the weight of the ball. But as this cannot be the caſe in practice, the center of oſcillation will always be further from the point of ſuſpenſion than the calculation makes it; and this muſt be found by trial.

To divide the Circumference of a Circle into any given Number of equal Parts, whether even or odd.

As there are very uncommon and odd numbers of teeth in ſome of the wheels of aſtronomical clocks, and which conſequently could not be cut by any common engine uſed by clock-makers for cutting the numbers of teeth in their clock-wheels, I thought proper to ſhew how to divide the circumference of a circle into any given odd or even number of equal parts, ſo as *that* number may be laid down upon the dividing-plate of a cutting engine.

There is no odd number, but from which, if a certain number be ſubtracted, there will remain an even number, eaſy to be ſubdivided. Thus, ſuppoſing the given number of equal diviſions of a circle on the dividing plate to be 69; ſubtract 9, and there will remain 60.

Every circle is ſuppoſed to contain 360 degrees: therefore ſay, As the given number of parts in the circle, which is 69, is to 360 degrees, ſo is 9 parts to the correſponding arc of the circle that will contain them: which arc, by the Rule of Three, will be found to be $46\frac{95}{100}$. Therefore, by the line of chords on a common ſcale, or rather on a ſector, ſet off $46\frac{95}{100}$ (or $46\frac{9}{10}$) degrees with your compaſſes, in the periphery of the circle, and divide that arc or portion of the circle into 9 equal parts, and the reſt of the circle into 60; and the whole will be divided into 69 equal parts, as was required.

Again, ſuppoſe it were required to divide the circumference of a circle into 83 equal parts; ſubtract 3, and 80 will remain.—Then, as 83 parts are to 360 degrees, ſo (by the Rule of Proportion) are 3 parts to 13 degrees and one hundredth part of a degree; which ſmall fraction may be neglected. Therefore, by the line of chords, and compaſſes, ſet off 13 degrees in the periphery of the circle, and divide that portion or arc into 3 equal parts, and the reſt of the circle into 80; and the thing will be done.

Once more, ſuppoſe it were required to divide a given circle into 365 equal parts: ſubtract 5, and 360 will remain. Then, as 365 parts are to 360 degrees, ſo are 5 parts to $4\frac{93}{100}$ degrees. Therefore, ſet off $4\frac{93}{100}$ (or $4\frac{9}{10}$) degrees in the circle; divide that ſpace into 5 equal parts, and the reſt of the circle into 360; and the whole will be divided into 365 equal parts, as was required.

I have often found this rule or method very uſeful in dividing circles into an odd number of equal parts, or wheels into odd numbers of equal ſiz'd teeth with equal ſpaces between them: and now I find it juſt as eaſy to divide any given circle into any odd number of equal parts, as to divide it into any even number. And, for this purpoſe, I prefer the line of chords on a ſector to that on a plain ſcale; becauſe the ſector may be opened ſo, as to make the radius of the line of chords upon it equal to the radius of the given circle, unleſs the radius of the circle exceeds the whole length of the ſector when it is opened ſo as to reſemble a ſtraight ruler or ſcale; and this is what very ſeldom happens.

Any perſon who is uſed to handle the compaſſes, and the ſcale or ſector, may very eaſily, by a little practice, take off degrees, and fractional parts

of

of a degree, by the accuracy of his eye, from a line of chords, near enough the truth for the above-mentioned purpoſe.

Suppoſing the diſtance between the Centers of two Wheels, one of which is to turn the other, be given; that the Number of Teeth in one of theſe Wheels is different from the Number of teeth in the other, and it is required to make the Diameters of theſe Wheels in ſuch Proportion to one another as their Numbers of Teeth are, ſo that the Teeth in both Wheels may be of equal Size, and the Spaces between them equal, that either of them may turn the other eaſily and freely: it is required to find their Diameters.

Here it is plain that the diſtance between the centers of the wheels is equal to the ſum of both their radii in the working parts of the teeth.—

Therefore, as the number of teeth in both wheels, taken together, is to the diſtance between their centers, taken in any kind of meaſure, as feet, inches, or parts of an inch; ſo is the number of teeth in either of the wheels to the radius or ſemidiameter of that wheel, taken in the like meaſure, from its center to the working part of any one of its teeth.

Thus, ſuppoſe the two wheels muſt be of ſuch ſizes, as to have the diſtance between their centers 5 inches; that one wheel is to have 75 teeth, and the other to have 33, and that the ſizes of the teeth in both the wheels is equal, ſo that either of them may turn the other. The ſum of the teeth in both wheels is 108; therefore ſay, As 108 teeth is to 5 inches, ſo is 75 teeth to $3\frac{47}{100}$ inches; and as 108 is to 5, ſo is 33 to $1\frac{53}{100}$. So that, from the center of the wheel of 75 teeth to the working part of any tooth in it, is 3 inches and 47 hundred parts of an inch; and, from the center of the wheel of 33 teeth to the working part of either of its teeth, is 1 inch and 53 hundred parts of an inch.

The Deſcription and Uſe of a New Machine called the MECHANICAL PARADOX.

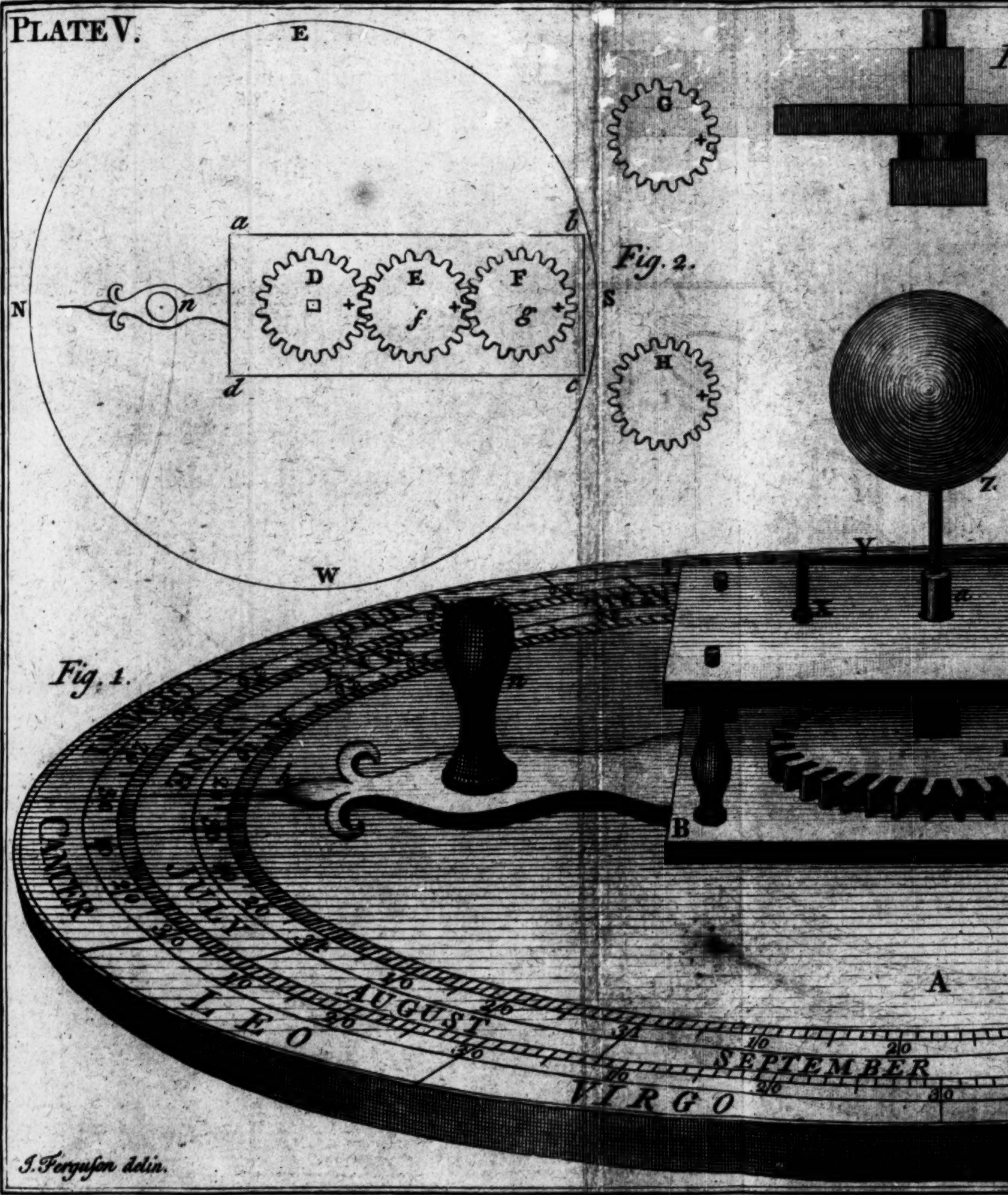
The vulgar and illiterate take almoſt every thing, even the moſt important, upon the authority of others, without ever examining it themſelves.—Although this implicit confidence is ſeldom attended with any bad conſequences in the common affairs of life, it has nevertheleſs, in other things, been much abuſed; and in political and religious matters, has produced fatal effects. On the other hand, knowing and learned men, to avoid this weakneſs, have fallen into the contrary extreme: ſome of them believe every thing to be unreaſonable, or impoſſible, that appears ſo to their firſt apprehenſion; not adverting to the narrow limits of the human underſtanding, and the infinite variety of objects, with their mutual operations, combinations, and affections, that may be preſented to it.

It muſt be owned, that credulity has done much more miſchief in the world than incredulity has done, or ever will do; becauſe the influences of the latter extend only to ſuch as have ſome ſhare of education, or affect the reputation thereof.—And ſince the human mind is not neceſſarily impelled, without evidence, either to belief or unbelief; but may ſuſpend its aſſent to, or diſſent from, any propoſition till after a thorough examination; it is to be wiſhed, that men of literature, eſpecially philoſophers, would not haſtily, and by firſt appearances, determine themſelves with reſpect to the truth or falſehood, poſſibility or impoſſibility of things.

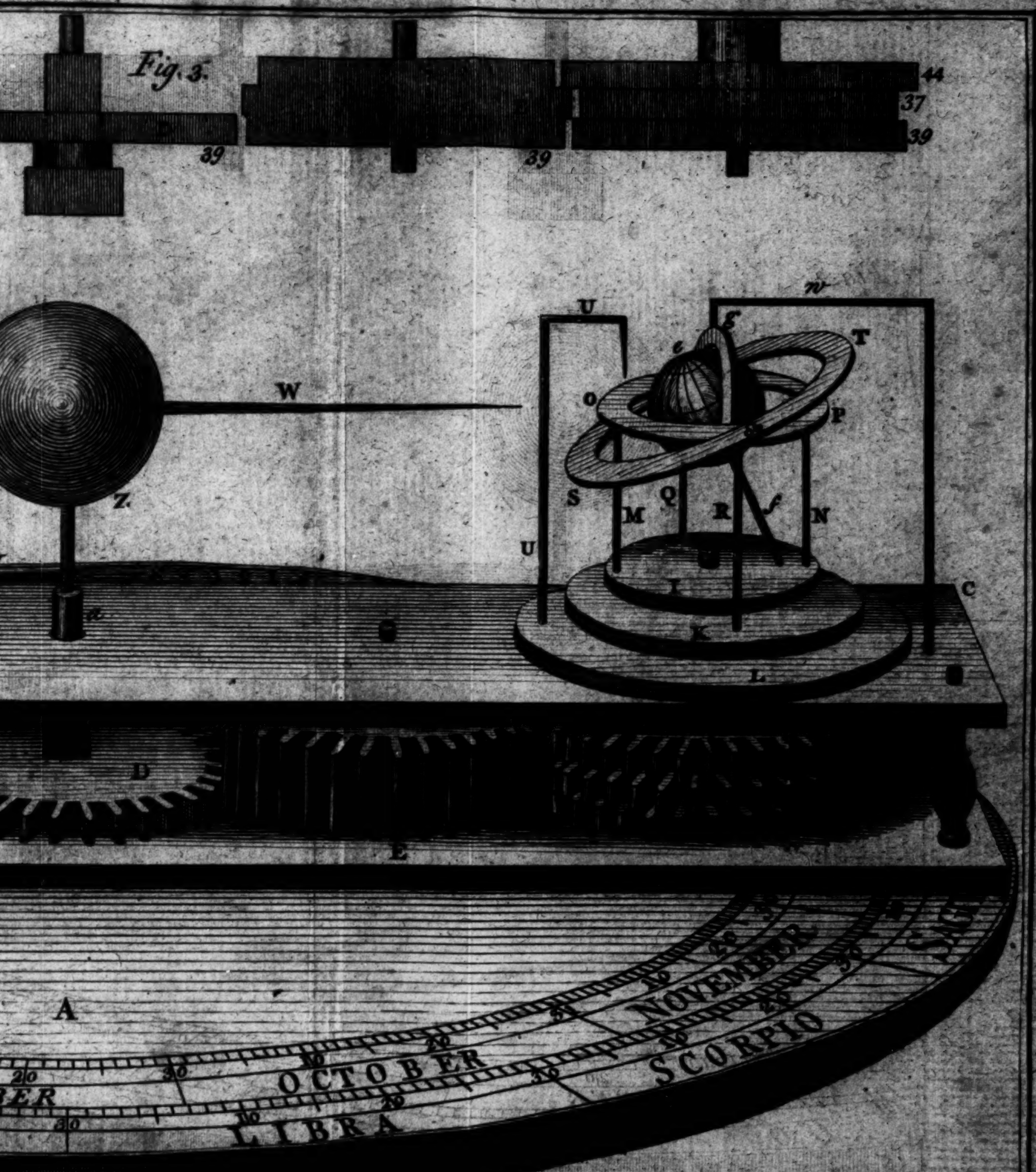
A perſon who has made but little progreſs in the mathematics, though in other reſpects learned and judicious, would be apt to pronounce it impoſſible that two lines, which were no where two inches aſunder, may continually approach toward one another, and yet never meet, although continued to infinity: and yet the truth of this propoſition may be eaſily demonſtrated.—And many, who are good mechanics, would be as apt to pronounce the ſame, if they were told, that although the teeth of one wheel ſhould take equally deep into the teeth of three others, it ſhould affect them in ſuch a manner, that in turning it any way round its axis, it ſhould turn one of them *the ſame way*, another *the contrary way*, and the third *no way at all.*

On a very particular occaſion, about eighteen years ago, I contrived a ſmall machine of this ſort, which has been ſhewn

PLATE V.



J. Ferguson delin.



J. Lodge sculp.

ſhewn and explained to many; and which I ſhall here deſcribe, and explain ſome of the uſes it has been applied to.

It is repreſented to view by Fig. 1. of Plate V. in which *A* is called *the immoveable plate*, becauſe it lies ſtill on a table whilſt the machine is at work. *B C* is a moveable frame, to be turned round an upright axis *a* (fixed into the center of the immoveable plate) by taking hold of the knob *n*, which is fixed into the index *h*.

On the ſaid axis is fixed the immoveable wheel *D*, whoſe teeth take into the teeth of the thick moveable wheel *E*, and turn it round its own axis, as the frame is turned round the fixed axis of the immoveable wheel *D*; and in the ſame direction that the frame is moved.

The teeth of the thick wheel *E* take equally deep into the teeth of the three wheels *F*, *G*, and *H*; but operate on theſe wheels in ſuch a manner, that whilſt the frame is turned round, the wheel *H* turns *the ſame way* that the wheel *E* does; the wheel *G* turns *the contrary way*, and the wheel *F* turns *no way* at all.

Before we explain the principles on which theſe three different effects depend, it will not be improper to fix ſome certain *criteria* for bodies turning or not turning round their own axes or centers; and to make a diſtinction between abſolute and relative motion.

1. If a body ſhews all its ſides progreſſively round toward a certain fixed point in the heavens, the body turns round its own axis or center, whether it remains ſtill in the ſame place, or has a progreſſive motion in any orbit whatever.—For, unleſs it does turn round its own center, it cannot poſſibly have one of its ſides toward the weſt at one time, toward the ſouth at another, toward the eaſt at a third time, and toward the north at a fourth.—

This is the caſe with the Moon, which always keeps one ſide toward the earth; but ſhews the ſame ſide to every fixed point of the ſtarry heaven in the plane of her orbit, in the time ſhe goes once round her orbit; becauſe in the time that ſhe goes round her orbit, ſhe turns once round her own axis or center.—On the contrary, if a body ſtill keeps one of its ſides toward a fixed point of the heaven, the body does not turn round its own axis or center, whether it keeps in one and the ſame place or has a progreſſive motion in any orbit or direction whatever.—This is the caſe with the card of the compaſs in a ſhip, which ſtill keeps one of its points toward the magnetic north, let the ſhip be at reſt, or ſail round a circle of many miles diameter.

Both theſe caſes may be exemplified either by a cube or a globe, having a pin fixed into either of its ſides to hold it by: we ſhall ſuppoſe a cube, becauſe its ſides are flat.—Sit down at a table, and hold the cube by the pin, which may be called its axis, and keep one of its ſides toward any ſide of the room. Whilſt you do this, you do not turn the cube round its axis, whether you ſtill keep it in the ſame place, or carry it round any other fixed body on the table.—But if you try to keep any ſide of the cube toward the fixed body, whilſt you are carrying it round the ſame, you will find that you cannot do ſo, without turning the pin round (which is fixed into the cube) betwixt the finger and thumb whereby you hold it; unleſs you riſe and walk round the table, keeping your face always toward the fixed body on the table; and then, both yourſelf and the cube will have turned once round; for the cube will have ſhewn the ſame ſide progreſſively round to all ſides of the room, and your face will have been turned toward every ſide of the room, and every fixed point of the horizon.

2. If a ſhip turns round, and at the ſame time a man ſtands on the deck without moving his feet, he is turned abſolutely round by the motion of the ſhip, though he has no relative motion with reſpect to the ſhip.—But if, whilſt the ſhip is turning round, he endeavours to turn himſelf round the contrary way; he thereby only undoes the effect that the turning of the ſhip would otherwiſe have had upon himſelf; and is, in fact, ſo far from turning abſolutely round, that he keeps himſelf from turning at all; and the ſhip turns round him, as round a fixed axis; although, with reſpect to the ſhip, he has a relative motion.

Fig. 2. is a ſmall plan, or flat view of the machine, in which, the ſame letters of reference are put to the wheels in it, as to thoſe in Fig. 1. for the conveniency of looking at both the figures, in reading the deſcription of them. *WSEN* is the round immoveable plate: *D* the immoveable wheel on the fixed axis in the center of that plate: *E* the thick moveable wheel, whoſe teeth take into the teeth of the wheel *D*; and *F* is one of the thin wheels, over which *G* and *H* may be put; and then, *F*, *G*, and *H* will make a thickneſs equal to the thickneſs of the wheel *E*, and its teeth will take equally deep into the teeth of them all. The frame that holds theſe wheels is repreſented by the parallelogram *a b c d*; and if it be turned round, it can give no motion to the wheel *D*, becauſe that wheel is fixed on an axis which is fixed into the great immoveable plate.

Take away the thick wheel *E*, and leave the wheel *F* where it lies, on the lower plate of the frame. Then turn the frame round the axis of the immoveable plate *WSEN* (denoted by *A* in Fig. 1.) and it will carry the wheel *F* round with it.—In doing this, *F* will ſtill keep one and the ſame ſide toward the fixed central wheel *D*, as the Moon ſtill keeps the ſame ſide toward the Earth: and although *F* will then have no relative motion with reſpect to the moving frame, it will be abſolutely turned round its own center *g* (like the man on the ſhip whilſt he ſtood without moving his feet on the deck), for the croſs mark on its ſide next *S* will be progreſſively turned toward all the ſides of the room.

But, if we would keep the wheel *F* from turning round its own center, and ſo cauſe the croſs mark upon it to keep always toward one ſide of the room; or, like the magnetic needle, to keep the ſame point ſtill toward one fixed point in the horizon; we muſt produce an effect upon *F*, reſembling what the man on the ſhip did, by endeavouring to turn himſelf round the contrary way to that which the ſhip turned, ſo as he might keep from turning at all; and by that means keep his face ſtill toward one and the ſame point of the horizon.—And this is done, by making the numbers of teeth equal in the wheels *D* and *F* (ſuppoſe 20 in each) and putting the thick wheel *E* between them, ſo as to take into the teeth of them both. For then, as the frame is turned round the axis of the fixed wheel *D*, by means of the knob *n*, the wheel *E* is turned round its axis by the wheel *D*; and, for every ſpace of a tooth that the frame would turn the wheel *F*, in direction of the motion of the frame, the wheel *E* will counteract that motion, by turning the wheel *F* juſt as far backward with reſpect to the motion of the frame; and ſo will keep *F* from turning any way round its own center: and the croſs mark near its edge will be always directed towards one ſide of the room.—Whether the wheel *E* has the ſame number of teeth as *D* and *F* have, or any different number, its effect on *F* will be ſtill the ſame.

If *F* had one tooth leſs in number than *D* has, the effect produced on *F*, by the turning of the frame, would be as much more than counteracted by the intermediate wheel *E*, as is equal to the ſpace of one tooth in *F:* and therefore whilſt the frame was turned once round, ſuppoſe in direction of the letters *W S E N* on the immoveable plate, the wheel *F* would be turned the contrary way, as much as is equal to the ſpace taken up by one of its teeth. But, if *F* had one tooth more in number than *D* has, the effect of the motion of the frame (which is to turn *F* round in the ſame direction with it) would not be fully counteracted by means of the intermediate wheel *E*; for as much of that effect would remain as is equal to the ſpace of one tooth in *F:* and therefore, in the time the frame was turned once round, the wheel *F* would turn, on its own center, in direction of the motion of the frame, as much as is equal to the ſpace taken up by one of its teeth: and here note, that the wheel *E* (which turns *F)* always turns in direction of the motion of the frame.

And therefore, if an upright pin be fixed into the lower plate of the frame, under the center of the wheel *F*, and if the wheel *F* has the ſame number of teeth that the fixed wheel *D* has, the wheel *G* one tooth leſs, and the wheel *H* one tooth more; and if theſe three wheels are put looſely upon this pin, ſo as to be at liberty to turn either way; and the thick wheel *E* takes into the teeth of them all, and alſo into the teeth of the fixed wheel *D*; then, whichever way the frame is turned, the wheel *H* will turn *the ſame way*, the wheel *G the contrary way*, and the wheel *F no way* at all.—The leſs number of teeth *G* has, with reſpect to thoſe of *D*, the faſter it will turn backward; and the greater number of teeth *H* has, with reſpect to thoſe in *D*, the faſter it will turn forward; reckoning *that* motion to be backward which is contrary both to the motion of the frame and of the thick wheel *E*, and *that* motion to be forward which is in the ſame direction with the motion of the frame and of the wheel *E*.—So that the turning or not turning of the three wheels, *F*, *G*, *H*, or the direction and velocity of the motions of thoſe that do turn round, depends entirely on the relation between their numbers of teeth and the number of teeth in the fixed wheel *D*, without any regard to the number of teeth in the moveable wheel *E*.

Having ſolved the paradox, and deſcribed the cauſe of the different effects which are produced upon the three wheels F, G, *and* H, *we ſhall now proceed to ſhew ſome uſes that may be made of the machine.*

This machine is ſo much of an ORRERY, as is ſufficient to ſhew the different lengths of days and nights, the viciſſitudes of the ſeaſons, the retrograde motion of the nodes of the Moon's orbit, the direct motion of the apogeal point of her orbit, and the months in which the Sun and Moon muſt be eclipſed.

On the great immoveable plate *A* (ſee Fig. 1.) are the months and days of the year, and the ſigns and degrees of the zodiac ſo placed, that when the annual index *h* is brought to any given day of the year, it will point to the degree of the ſign in which the Sun is on that day.—This index is fixed to the moveable frame *B C*, and is carried round the immoveable plate with it, by means of the knob *n*. The carrying this frame and index round the immoveable plate, anſwers to the Earth's annual motion round the Sun, and to the Sun's apparent motion round the ecliptic in a year.

The central wheel *D* (being fixed on the axis *a*, which is fixed in the center of the immoveable plate) turns the thick wheel *E* round its own axis by the motion of the frame; and the teeth of the wheel *E* take into the teeth of the three wheels *F*, *G*, *H*, whoſe axes turn within one another, like the axes of the hour, minute, and ſecond hands of a clock or watch, where the ſeconds are ſhewn from the center of the dial-plate.

On the upper ends of theſe axes are the round plates *I*, *K*, *L*; the plate *I* being on the axis of the wheel *F*, *K* on the axis of *G*, and *L* on the axis of *H*. So that, whichever way theſe wheels are affected, their reſpective plates, and what they ſupport, muſt be affected in the ſame manner; each wheel and plate being independent of the others.

The two upright wires *M* and *N* are fixed into the plate *I*; and they ſupport the ſmall ecliptic *O P*, on which, in the machine, the ſigns and degrees of the ecliptic are marked.—This plate alſo ſupports the ſmall terreſtrial globe *e* on its inclining axis *f*, which is fixed into the plate near the foot of the wire *N*. This axis inclines 23½ degrees from a right line, ſuppoſed to be perpendicular to the ſurface of the plate *I*, and 66½ to the plane of the ſmall ecliptic *O P* which is parallel to that plate.

On the Earth *e* is the creſcent *g*, which goes more than half way round the Earth, and ſtands perpendicular to the plane of the ſmall ecliptic *O P*, directly facing the Sun *Z*: its uſe is to divide the enlightened half of the Earth next the Sun from the other half which is then in the dark; ſo that it repreſents the boundary of light and darkneſs, and therefore ought to go quite round the Earth; but cannot, in a machine, becauſe, in ſome poſitions, the Earth's axis would fall upon it.—The Earth may be freely turned round on its axis by hand, within the creſcent, which is ſupported by the crooked wire *w*, fixed to it, and into the upper plate of the moveable frame *B C*.

In the plate *K* are fixed the two upright wires *Q* and *R*: they ſupport the Moon's inclined orbit *S T* in its nodes, which are the two oppoſite points of the Moon's orbit where it interſects the ecliptic *O P*. The aſcending node is marked ☊, to which the deſcending node is oppoſite, below *e*, but hid from view by the globe *e*. The half ☊ *T e* of this orbit is on the north ſide of the ecliptic *O P*, and the other half *e S* ☊ is on the ſouth ſide of the ecliptic. The Moon is not in this machine: but, when ſhe is in either of the nodes of her orbit in the heavens, ſhe is then in the plane of the ecliptic: when ſhe is at *T* in her orbit, ſhe is in her greateſt north latitude; and when ſhe is at *S*, ſhe is in her greateſt ſouth latitude.

In the plate *L* is fixed the crooked wire *U U*, which points downward to the ſmall ecliptic *O P*, and ſhews the motion of the Moon's apogee therein, and its place at any given time.

The ball *Z* repreſents the Sun, which is ſupported by the crooked wire *X Y*, fixed into the upper plate of the frame at *X*. A ſtraight wire *W* proceeds from the Sun *Z*, and points always toward the center of the Earth *e*; but toward different points of its ſurface at different times of the year, on account of the obliquity of its axis, which keeps its paralleliſm during the Earth's annual courſe round the Sun *Z*; and therefore muſt incline ſometimes toward the Sun, at other times from him, and twice in the year neither toward nor from the Sun, but ſidewiſe to him. The wire *W* is called *the ſolar ray*.

As the annual index *h* ſhews the Sun's place in the ecliptic for every day of the year, by turning the frame round the axis of the immoveable plate *A*, according to the order of the months and ſigns, the ſolar ray does the ſame in the ſmall ecliptic *O P*: for, as this ecliptic has no motion on its axis, its ſigns and degrees ſtill keep parallel to thoſe on the immoveable plate. At the ſame time, the nodes of the Moon's orbit *S T* (or points where it interſects the ecliptic *O P*) are moved backward, or contrary to the order of ſigns, at the rate of $19\frac{1}{3}$ degrees every *Julian* year; and the Moon's apogeal wire *U U* is moved forward, or according to the order of the ſigns of the ecliptic, nearly at the rate of 41 degrees every *Julian* year; the year being denoted by a revolution of the Earth *e* round the Sun *Z*; in which time the annual index *h* goes round the circles of months and ſigns on the immoveable plate *A*.

Take hold of the knob *n*, and turn the frame round thereby; and in doing this, you will perceive that the north pole of the Earth *e* is conſtantly before the creſcent *g*, in the enlightened part of the Earth toward the Sun, from the 20th of March to the 23d of September; and the ſouth pole all that time behind the creſcent in the dark: and, from the 23d of September to the 20th of March, the north pole is conſtantly in the dark, behind the creſcent, and the ſouth pole in the light before it:

which ſhews that there is but one day and one night at each pole, in the whole year; and that, when it is day at either pole, it is night at the other.

From the 20th of March to the 23d of September, the days are longer than the nights in all thoſe places of the northern hemiſphere of the Earth which revolve through the light and dark, and ſhorter in thoſe of the ſouthern hemiſphere.—From the 23d of September to the 20th of March, the reverſe.

There are 24 meridian ſemicircles drawn on the globe, all meeting in its poles; and as one rotation or turn of the Earth on its axis is performed in 24 hours, each of theſe meridians is an hour diſtant from the other, in every parallel of latitude.—Therefore, if you bring the annual index *h* to any given day of the year, on the immoveable plate, you may ſee how long the day then is at any place of the Earth, by counting how many of theſe meridians are in the light, or before the creſcent, in the parallel of latitude of that place; and this number being ſubtracted from 24 hours, will leave remaining the length of the night.—And if you turn the Earth round its axis, all thoſe places will paſs directly under the point of the ſolar ray, which the Sun paſſes vertically over on that day, becauſe they are juſt as many degrees north or ſouth of the equator, as the Sun's declination is then from the equinoctial.

At the two equinoxes, *viz.* on the 20th of March and 23d of September, the Sun is in the equinoctial, and conſequently has no declination. On theſe days, the ſolar ray points directly toward the equator, the Earth's poles lie under the inner edge of the creſcent, or boundary of light and darkneſs; and, in every parallel of latitude, there are twelve of the meridians, or hour-circles before the creſcent, and twelve behind it; which ſhews that the days and nights then are each twelve hours long at all places of the Earth. And, if the Earth be turned round its axis, you will ſee that all places on it go equally through the light and the dark hemiſpheres.

On the 21ſt of June, the whole ſpace within the north polar circle is enlightened, which is 23½ degrees from the pole, all around; becauſe the Earth's axis then inclines 23½ degrees toward the Sun; but the whole ſpace within the ſouth polar circle is in the dark; and the ſolar ray points toward the tropic of Cancer on the Earth, which is 23½ degrees north from the equator.—On the 20th of December the reverſe happens, and the ſolar ray points toward the tropic of Capricorn, which is 23½ degrees ſouth from the equator.

If you bring the annual index *h* to the beginning of January, and turn the Moon's orbit *S T* by its ſupporting wires *Q* and *R* till the aſcending node (marked ☊) comes to its place in the ecliptic *O P*, as found by an Ephemeris, or by Aſtronomical Tables, for the beginning of any given year; and then move the annual index by means of the knob *n*, till the index comes to any given day of the year afterward, the nodes will ſtand againſt their places in the ecliptic on that day.—And if you move the index onward, till either of the nodes comes directly againſt the point of the ſolar ray, the index will then be at the day of the year on which the Sun is in conjunction with that node. At the times of thoſe new Moons which happen within ſeventeen days of the conjunction of the Sun with either of the nodes, the Sun will be eclipſed: and at the times of thoſe full Moons, which happen within twelve days of either of theſe conjunctions, the Moon will be eclipſed.—Without theſe limits there can be no eclipſe either of the Sun or Moon; becauſe, in nature, the

Moon's latitude or declination from the ecliptic, is too great for the Moon's ſhadow to fall on any part of the Earth, or for the Earth's ſhadow to touch the Moon.

Bring the annual index to the beginning of January, and ſet the Moon's apogeal wire *U U* to its place in the ecliptic for that time, as found by Aſtronomical Tables: then move the index forward to any given day of the year, and the wire will point on the ſmall ecliptic to the place of the Moon's apogee for that time.

The Earth's axis *f* inclines always toward the beginning of the ſign Cancer on the ſmall ecliptic *O P*.—And, if you ſet either of the Moon's nodes, and her apogeal wire, to the beginning of that ſign, and turn the plate *A* about, until the Earth's axis inclines toward any ſide of the room (ſuppoſe the north ſide) and then move the annual index round and round the immoveable plate *A*, according to the order of the months and ſigns upon it, you will ſee that the Earth's axis and beginning of Cancer will ſtill keep toward the ſame ſide of the room, without the leaſt deviation from it; but the nodes of the Moon's orbit *S T* will turn progreſſively towards all the ſides of the room, contrary to the order of ſigns in the ſmall ecliptic *O P*, or from eaſt, by ſouth, to weſt, and ſo on: and the apogeal wire *U U* will move the contrary way to the motion of the nodes, or according to the order of the ſigns in the ſmall ecliptic, from weſt, by ſouth, to eaſt, and ſo on quite round.—A clear proof that the wheel *F*, which governs the Earth's axis and the ſmall ecliptic, does not turn any way round its own center; that the wheel *G*, which governs the Moon's orbit *O P*, turns round its own center backward, or contrary both to the motion of the frame *B C* and thick wheel *E*; and that the wheel *H*, which governs the Moon's apogeal wire *U U*, turns round its own center, forward, or in direction both of the motion of the frame, and of the thick wheel *E*, by which the three wheels, *F*, *G*, and *H*, are affected.

The wheels *D*, *E*, and *F*, have each 39 teeth in the machine; the wheel *G* has 37, and *H* 44; as ſhewn in Fig. 3.

The parallelíſm of the Earth's axis is perfect in this machine; the motion of the apogee very nearly ſo; the motion of the nodes not quite ſo near the truth, though they will not vary ſenſibly therefrom in one year.—But they cannot be brought nearer, unleſs larger wheels, with higher numbers of teeth, are uſed.

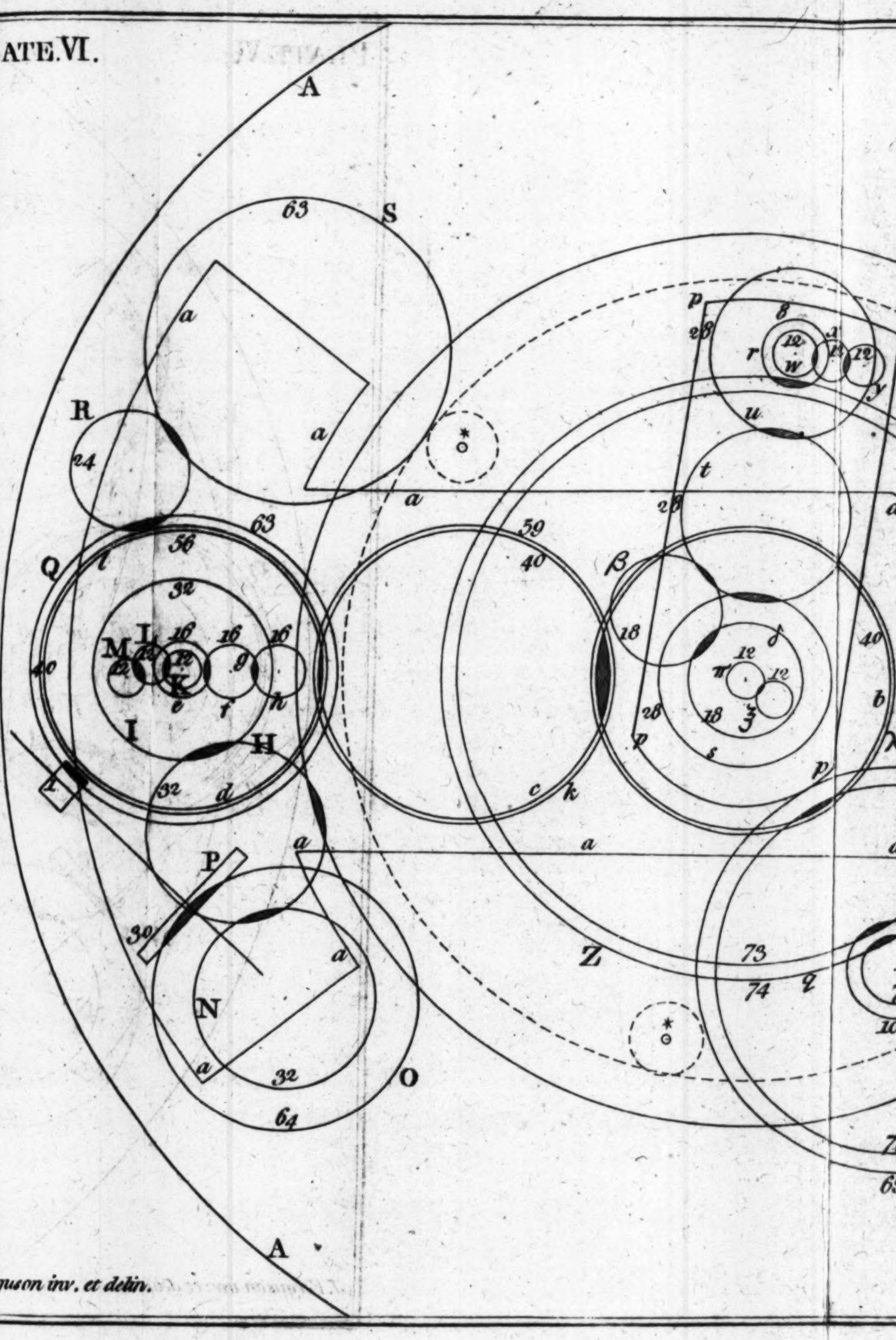
In nature, the Moon's apogee goes quite round the ecliptic in eight years and 312 days, in direction of the Earth's annual motion; and the nodes go round the ecliptic in a contrary direction, in eighteen years and 225 days.—In the machine, the apogee goes round the ecliptic *O P* in eight years and four-fifths of a year, and the nodes in eighteen years and a half.

Notwithſtanding the difference of the numbers of teeth in the wheels *F*, *G*, and *H*, and their being all of equal diameters, they take tolerably well into the teeth of the thick wheel *E*, becauſe they are made of ſoft wood. —But, if they were made of metal, the wheel *E* in Fig. 1. ought to be made of the ſhape of *E* (ſeen edgewiſe) in Fig. 3. with very deep teeth: and the wheels *F*, *G*, and *H*, in Fig. 1. of diameters proportioned to their reſpective numbers of teeth, as *F*, *G*, and *H*, in Fig. 3. And then the teeth of theſe three wheels would be of equal ſizes with thoſe of the wheel *E* wherein they work: and the motions would be free and eaſy, without any pinching or ſhake in the teeth.

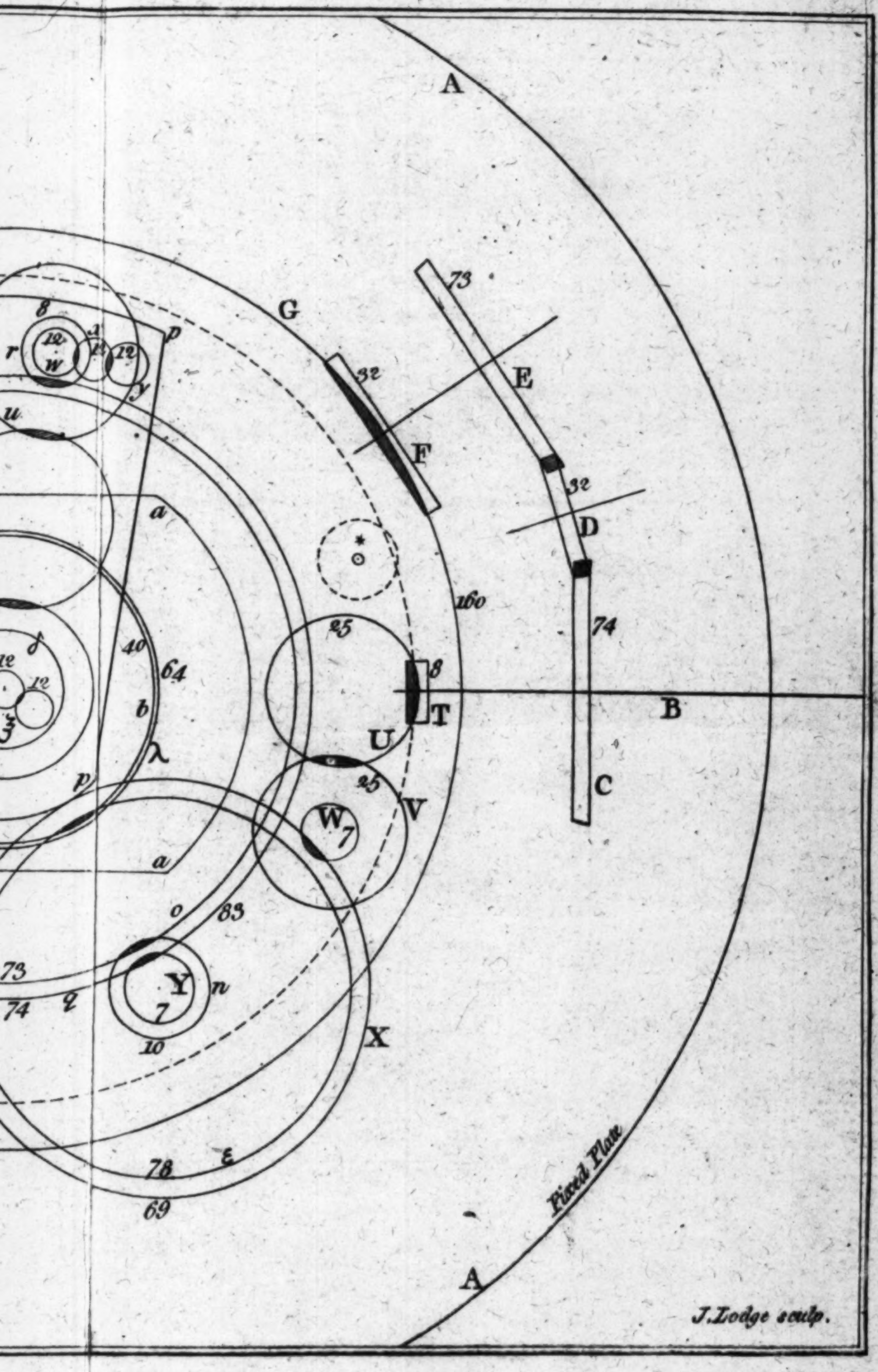
An Orrery, ſhewing the Motions of the Sun, Mercury, Venus, Earth, Moon, and Nodes of the Moon's Orbit; the different Lengths of Days and Nights, the Viciſſitudes of Seaſons, Age and Phaſes of the Moon, and all the Solar and Lunar Eclipſes.

The uſe of this Orrery, and the manner of uſing it, being already deſcribed in my Aſtronomy, I ſhall not repeat thoſe matters here; but ſhall only deſcribe the wheel-work of it, which is not done in that book. It is not copied from any other Orrery whatever, and I can truly ſay, that it ſhews the revolutions of the Moon and Planets nearer the truth than any other Orrery does, that has fallen under my examination. I therefore freely give the following account of it to the Public, in the beſt manner that I can; and do wiſh the deſcription may be generally underſtood. To

PLATE.VI.



J.Ferguson inv. et delin.



J. Lodge sculp.

any Clock-maker I hope it will be plain, and to every Orrery-maker I believe it will be quite ſo.

The ſixth Plate is a plan of the wheel-work, in which the diameter of each wheel is equal to the ſemidiameter or radius thereof in the Orrery I have made: the numeral figures at each wheel ſhew the number of teeth in that wheel, and the ſhaded parts ſhew where the teeth of any one wheel takes into the teeth of another, as the one turns the other.

The ſeventh Plate is a ſection or ſide-view of all the wheel-work that could be brought into ſight. But in this, ſome few wheels could not be ſhewn; for, in the Orrery itſelf, take a view of the wheels on any ſide you pleaſe, ſome of them will be unavoidably hid from ſight by others that are between them and the eye.

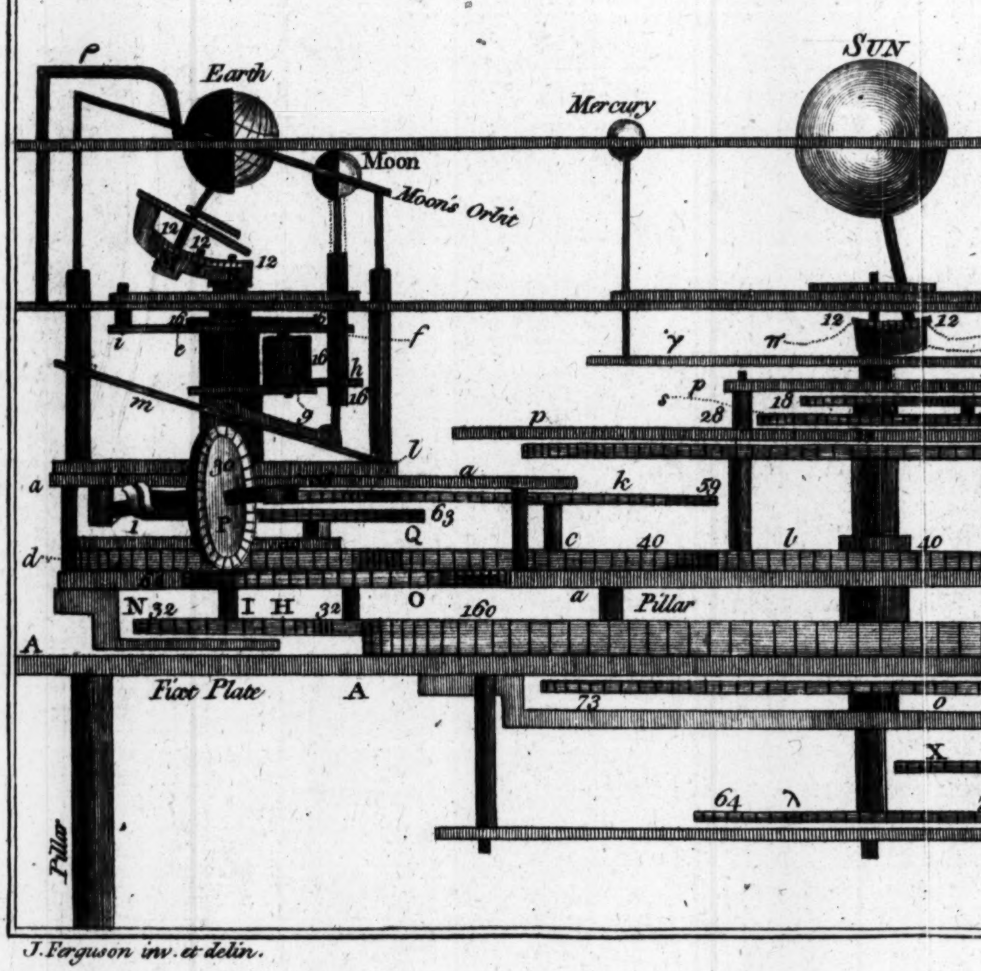
Thoſe in Plate VII. that come in ſight have the ſame numeral figures ſet to them as the like ones have in Plate VI.

and alſo the ſame letters of reference where there is room to inſert them. And therefore, in reading the deſcription of Plate VI. it will be requiſite to look firſt at it, and then at Plate VII.; by which means the Reader will ſee the poſition of theſe wheels with reſpect to each other, as they are placed higher or lower in the frames which contain them.

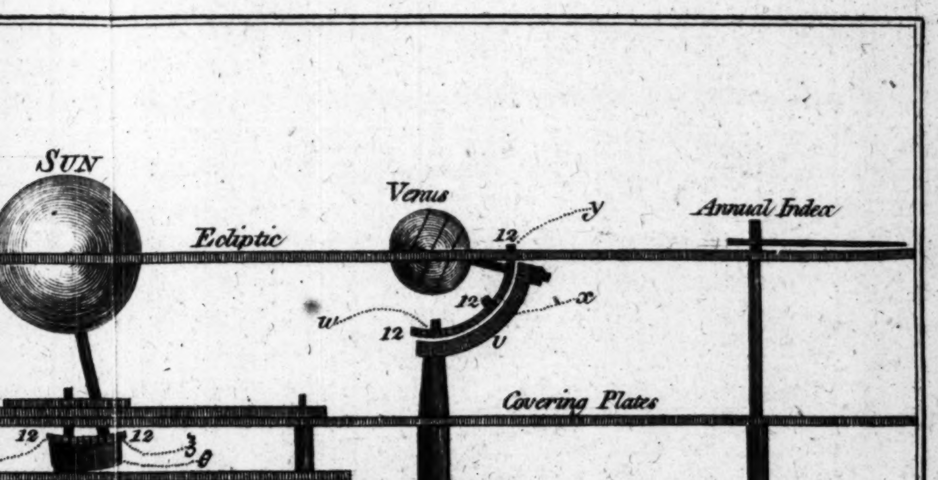
AAAA is a round immoveable plate ſupported by four pillars; ſome of the wheels are below it, but the greateſt number of them are above it. It ſupports and bears the weight of them all.

B is the axis of the handle or winch by which all the wheels are turned: on its axis is a wheel *C* of 74 teeth, which turns a wheel *D* of 32, and *D* turns a wheel *E* of 73 teeth, on whoſe axis is a wheel *F* of 32, turning a wheel *G* of 160 teeth, which turns a wheel *H* of 32, and *H* turns a wheel *I* of the ſame number, on the top of whoſe axis

PLATE VII.



J. Ferguson inv. et delin.



J. Lodge sculp.

is a ſmall wheel *K* of 12 teeth (juſt under the Earth) which turns a wheel *L* of the ſame number and ſize; and *L* turns ſuch another wheel *M* of the ſame number. The axis of *M* inclines 23½ degrees, and the Earth at the top of it is turned round by it. The wheel *H* of 32 teeth turns a wheel *N* of the ſame number, on the top of whoſe axis is an index which goes round a circle of 24 hours (on the plate that covers the wheel-work), in the time the Earth turns round its axis.—The wheels *D* and *E* could not be ſhewn in Plate VII. becauſe the wheel *C* of 74 teeth hides them from ſight.

On the axis of the wheel *N* is a wheel *O* of 64 teeth, turning a contrate wheel *P* of 30, on whoſe axis is an endleſs ſcrew, of a ſingle thread r, turning a wheel *Q* of 63 teeth, which carries the Moon round the Earth in her orbit, from change to change, in 29 days 12 hours 45 minutes. This wheel of 63 teeth turns a wheel *R* of

24, which turns a wheel *S* of 63 teeth round in 29 days 12 hours 45 minutes, on whoſe axis is an index that ſhews the days of the Moon's age on a circle of 29½ equal parts, on the plate that covers the wheel-work.

On *B*, the axis of the handle, is a pinion *T* of 8 leaves, turning a wheel *U* of 25 teeth, which turns another wheel *V* of the ſame number and ſize, on whoſe axis is a pinion *W* of 7 leaves, turning a wheel *X* of 69 teeth, on whoſe axis is a pinion *Y* of 7 leaves, turning a wheel *Z* of 83 teeth once round in 365 days 5 hours 48 minutes 57 ſeconds, and carrying the Earth round the Sun in that time. For, in this wheel are four ſhort pillars, whoſe upper ends are fixed into the lower plate of a moveable frame *a a a a a a a* (Plate VI.) that turns round on a fixed upright pin in the center of the plate *AAAA*, and contains the above-mentioned wheels belonging to the Earth and Moon: ſo that the whole frame goes round the center pin in the ſame time with the wheel *Z*.

This laſt wheel cannot be ſeen in Plate VII. becauſe it lies within the wheel *G*, which is only a thick ring having 160 teeth on its outſide. Its innermoſt ſide is repreſented by a dotted circle in Plate VI. and it is kept in its place by three rollers, marked * * *, which turn upon pins fixed in the great immoveable plate *A A A A*.

As the uppermoſt edge of the contrate wheel *F* (ſee Plate VII.) muſt come a little way through the plate *A A A A*, in order to turn the ring-wheel *G* that lies on the upper ſide of this plate, and this wheel turns the wheel *H* of 32 teeth that belongs to the Earth's diurnal motion; it is plain, that as the wheel *H* muſt go round *G* in a year by the annual part of the work, *G* muſt be thick enough to turn *H* at ſuch a diſtance from or above the plate *A A A A*, that *H* may go over the top of *F* without touching it: otherwiſe,

wiſe, when *H* came round to *F*, it could not paſs by, but would ſtop the annual motion.

In the center, juſt above the upper ſurface of the moveable frame-plate *aaaa*, is a fixed wheel *b* of 40 teeth taking into the teeth of the wheel *c*, which is alſo 40 in number; and theſe take into the teeth of a wheel *d*, whoſe number is 40 alſo. The axis of this laſt wheel is hollow, and the top of it is fixed tight at *K* (ſee Plate VII.) in the piece *K L M* that carries the Earth.—This part of the work keeps the Earth's inclined axis in a conſtant parallelism in its annual courſe round the Sun. For, as *d* is connected with the fixed wheel *b*, by means of the intermediate wheel *c*, and *c* rolls or goes round *b* by the annual work, and as *b*, *c* and *d* have equal numbers of teeth, *d* muſt always preſerve its parallelism throughout its annual motion. The axis of *b* is fixed into the immoveable plate *AAAA*; and it is hollow, to let

the

the axis of ſome wheels below that plate turn within it.

The ſolid ſpindle, or axis of the wheel *I* of 32 teeth, turns within the hollow axis of the wheel *d* of 40; and on the top of this ſolid ſpindle is the ſmall wheel *K* of 12 teeth, which turns the Earth round its axis by the wheels *L* and *M*, of equal number and ſize with *K*, as already mentioned.

The hollow axis of the paralleliſm-wheel *d* is within an upright ſocket, whoſe lowermoſt end is fixed into the top plate (marked 56 in Plate VII.) of the moveable frame *a a a a*, and on the top of this ſocket is fixed a ſmall wheel *e* of 16 teeth, which take into the teeth of another wheel *f* of the ſame number and ſize; on the axis of which is a long pinion *g* of 16 leaves, which take into the wheel *h* of 16 teeth, whoſe axis is hollow, and has a black cap on the top of it, covering juſt one half of the Moon.——Now, as the ſocket, on whoſe top the wheel *e* is placed, is fixed into the annual moving frame, it is plain, that, whichever ſide or tooth of the wheel *e* is once toward the Sun, will always be ſo; and therefore, as the wheel *f*, the pinion *g*, and the wheel *h*, go all round the wheel *e* by the work that carries the Moon round the Earth, and all theſe have equal numbers of teeth, the wheel *h* will always keep the Moon's cap facing towards the Sun, and ſhew her to be always full as ſeen from the Sun, but continually changing her phaſes as ſeen from the Earth in her going round it. For, when the Moon is between the Earth and the Sun (as repreſented in Plate VII.) her cap will hide the whole of her from the Earth: but, when ſhe is oppoſite to the Sun, all the half or ſide of her next the Earth will then appear like a full Moon, before the circular edge of the cap: and when ſhe is mid-way between theſe poſitions, or in either of her quadratures, ſhe will appear juſt half enlightened as ſeen from the Earth.

The axis of the wheel *Q*, of 63 teeth, which carries the Moon round the Earth, is hollow, and turns round upon the above-mentioned fixed ſocket. To the top of this axis (juſt under the wheel *e* of 16 teeth, Plate VII.) is fixed the bar *i f*, which carries the Moon round the Earth by the motion of the wheel *Q*.

On the top of the axis of the wheel *c* of 40 teeth, is a wheel *k* of 59, turning a wheel *l* of 56, which cauſe the nodes of the Moon's orbit to go once round, with a retrograde motion, thro' all the ſigns and degrees of the ecliptic, in $18\frac{2}{3}$ years. The axis of *l* is hollow, and turns upon the hollow axis of *Q*; and on the axis of *l* is a circular plate *m* (Plate VII.) fixed obliquely on that axis, and parallel to the Moon's orbit. The work that carries the Moon round the Earth carries alſo the piece *g* round upon this oblique plate; and, as the lower end of the Moon's axis (which turns within the hollow axis of her cap) is fixed into the piece *g*, it cauſes the Moon to riſe and fall in her oblique orbit, according to her north or ſouth latitude or declination from the ecliptic. As the nodes of her orbit are even with the plane of the ecliptic, one half of her orbit is on the north ſide, and the other half on the ſouth ſide of the ecliptic.

On the axis of the wheel *X*, which has 69 teeth, is a pinion *n* of 10 leaves, turning a wheel *o* of 73 teeth, which carries Venus round about the Sun in 224 days 17 hours. The axis of the wheel *o* is hollow (becauſe another axis turns within it) and on the top of it is fixed the lower plate of the frame *pppp*, which carries Venus round the Sun, and has wheels within it belonging to Venus and to Mercury.

Under the loweſt plate of this frame is a fixed wheel *q* of 74 teeth, of the ſame diameter as the wheel *Z* of 83, which gives the Earth its annual motion; ſo that, in Plate VI. one and the ſame circle repreſents both the wheels. —A pinion *r* of 8 leaves takes into the teeth of the fixed wheel *q*, and is carried round *q* by the motion of the frame *p p p p*, that carries Venus round the Sun. Conſequently, in the time this pinion is carried round the wheel, it will turn 9¼ times round its axis, equal to the number of Venus's days and nights in the time ſhe goes round the Sun.

The wheel *q* of 74 teeth is fixed on the ſame (above-mentioned) ſocket on which the wheel *b* of 40 teeth is fixed. The top of this ſocket goes through the lower plate of the frame *p p p p*, and a wheel *s* of 28 teeth is fixed upon the top of this ſocket, juſt above the ſame plate. Another wheel *t* of 28 teeth takes into the teeth of *s*, and is carried round it by the motion of the frame: and a third wheel *u* of 28 teeth (which is alſo carried round by the frame) takes into the teeth of *t*: the axis of *u* is hollow, it turns upon the ſolid ſpindle or axis of the pinion *r* of 8 leaves, and on its top is fixed the curved piece *v* (Plate VII.) that carries Venus on her inclined axis, which, by means of the three laſt-mentioned wheels of 28 teeth, is kept in a conſtant parallelíſm in going round the Sun.

On the top of the axis of the pinion *r* of 8 leaves, and juſt above the curved piece *v* (Plate VII.) is a ſmall wheel *w* of 12 teeth, which turns another wheel *x* of the ſame number and ſize; and this laſt wheel turns a third wheel *y* of the ſame number, which is fixed on the axis of Venus, and turns her 9¼ times round her axis in the time ſhe goes round the Sun; which is juſt as often as the pinion *r* turns round in the time it is carried round the fixed wheel *q* of 74 teeth.

On the top of the axis of the middle wheel *t* of 28 teeth, is another wheel of the ſame number and ſize, which turns a wheel β of 18 teeth; and this wheel turns another wheel δ of the ſame number, whoſe axis is a hollow ſocket, on which a bar γ (Plate VII.) is fixed; and this bar carries Mercury round the Sun in 87 days 23 hours.

On the axis of the wheel X (already mentioned) of 69 teeth is a wheel ε of 78, which turns a wheel λ of 64 round its axis in 25 days 6 hours. The axis of this wheel turns within the abovementioned hollow arbors in the center; and on its top is the ſmall wheel π of 12 teeth, which turns another wheel ξ of the ſame number and ſize: this laſt wheel is fixed on the Sun's axis, it turns in the fixed piece θ (Plate VII.) and turns the Sun round his axis in 25 days 6 hours.

The Sun's axis inclines $7\frac{1}{2}$ degrees from a perpendicular to the ecliptic; Venus's axis 75 degrees and the Earth's axis $23\frac{1}{2}$.

The Earth turns round within a black cap, that always covers the half of it which at any inſtant of time is turned quite away from the Sun: the edge of the cap repreſents the *ſolar horizon*, or circle bounding light and darkneſs: it is ſupported by a crooked wire ρ, whoſe lower end is fixed into the plate that covers the wheels, and is carried round by the annual motion-work. An index (called the *annual index*) goes round the ecliptic, by the ſame work, keeping always oppoſite to the Sun, and ſhewing the days of the months, and the Sun's apparent place in the ecliptic as ſeen from the Earth.

On looking at Plate VI. it may perhaps appear, even to a very ingenious mechanic, that the wheels *C*, *D*, and *E* are ſuperfluous; and that the wheel *F*, which gives motion to the toothed ring *G*, might have been upon the axis *B* of the handle. For, as *F* has 32 teeth, and *H*, that is turned by the teeth of *G* (and turns the Earth round its axis), has alſo 32 teeth, *F* and *H* would turn round in equal times; and conſequently, a turn of the handle would have anſwered to a turn of the Earth on its axis.—This indeed would have been the caſe if the Earth had no annual motion: but as *H* goes round *G* in a year, the ſame way that *G* turns round, *H* loſes five turns in going round *G* (for 5 times 32 is 160, the number of teeth in *G*), and then the handle would have turned 370 times round in the time the Earth made 365 rotations.—To prevent this, and ſo make the turns of the Earth and handle agree together, *C* has 74 teeth, and *E* only 73. So that the wheel *E* will turn five times oftener round than the handle does in 365 turns thereof; and conſequently make the Earth's daily rotation equal to a turn of the handle, or to 24 hours of mean ſolar time.

Another Orrery.

This is the Orrery mentioned in my *Tables and Tracts* (page 169, 2d edition) which I intended to keep for my ſon, who was then ſerving an apprenticeſhip to a mathematical inſtrument-maker. But, as it has pleaſed God to call him from this world to a better, I ſhall now freely communicate it to the Public.

It ſhews the length of day and night at all places of the Earth, every day of the year, with the Sun's true place, declination, time of riſing and ſetting, the hour of the day, the Sun's altitude, azimuth, and the variation of the compaſs at any place. Alſo the Moon's periodical and ſynodical revolution, her motion on her axis, her latitude, altitude, azimuth, riſing and ſetting; her mean anomaly and elliptic equation; with the days of all the new and full Moons and eclipſes, for 6000 years before and after the Chriſtian æra.—The out-ſide figure of this Orrery is exactly ſhewn in the eighth plate of my Aſtronomy; but the inſide work differs much from what is repreſented in the ſecond figure of that plate: and this inſide work is what I ſhall now deſcribe.

A large wheel of 235 teeth is fixed in the box that contains the work, the center of the wheel being in the center of the box, directly under the Sun's center. On this wheel runs a pinion of 19 leaves, carried round the teeth of the wheel by the annual motion of the Earth; and by this means, the pinion is turned round its own axis for every 19 teeth that it is carried onward, in going round the wheel.—Now, ſuppoſing this pinion to be carried round the wheel in 365¼ days, the pinion will be turned round its own axis in 29 days 12 hours 44 minutes 25 ſeconds, and a bar on the axis of the pinion will carry the Moon round the Earth, from change to change, in that time. This comes ſo near to the truth, as to vary but one day in the Moon's courſe in 335 years; and theſe are the neareſt numbers for ſuch ſimple wheel-work that can poſſibly be found for mean lunations.

But in nature, the Earth moves unequally round the Sun, ſo that there are 8 days more between the vernal and autumnal equinox, than between the autumnal and vernal.—And therefore, in common Orreries, where this circumſtance is taken no notice of, the Earth's poſition to the Sun cannot be right at both the equinoctial points.

In order to avoid this error, I firſt divided the ecliptic into 360 equal parts for degrees; and then, after having put the names of the ſigns to it, I laid down the days of the year from an ephemeris againſt the degrees of the Sun's place in the ecliptic, for each day reſpectively throughout the year. By this means, the daily ſpaces anſwering to the Earth's unequal motion round the Sun, were ſo divided, as to be continually and gradually leſſening from the 30th of December till the firſt of July; and then as gradually lengthening from the firſt of July to the 30th of December; as the Earth's progreſſive annual motion is ſwifteſt of all on the 30th of December, and ſloweſt of all on the firſt of July.

The days of the months being unequally divided, ſo as to anſwer to the Earth's unequal motion round the Sun, I made theſe diviſions a pattern or ſcale for dividing the 235 teeth of the above-mentioned wheel into ſuch unequal ſpaces as would agree with the ſpaces allotted for the days anſwering to them. But theſe gradual inequalities of the teeth were ſo very ſmall, and the difference ſo little between the wideſt and narroweſt, that the pinion (whoſe leaves were all equal, run very ſmoothly through all the teeth of the wheel; as the leaves of the pinion were ſized to theſe teeth, which were at a mean rate between the greateſt and leaſt diſtances from one another. By this contrivance, the mean lunation was always 29 days 12 hours 44 minutes 25 ſeconds throughout the whole year; and the pinion was among the leaſt diſtant teeth of the wheel when the annual index was at the firſt of July, and among the moſt diſtant teeth when the index was at the 30th of December.

For the parallelіſm of the Earth's axis, a wheel of 59 teeth was fixed in the middle of the work, with its center directly over the center of the wheel of 235 teeth, and the teeth of another wheel of 59 took into the teeth of the former; and thoſe into the teeth of a third wheel of the ſame number, on the top of whoſe axis the piece that carries the Earth on its oblique axis was fixed. And, as the Earth was moved round the Sun, theſe three wheels kept the parallelіſm of the Earth's axis, as deſcribed in the *Mechanical*

chanical Paradox, and the former Orrery.

Above the middlemoſt wheel of 59, and on its axis, is fixed a wheel of the ſame number, which takes into a wheel of 56 teeth: this laſt wheel is juſt below the Earth, and turns the nodes of the Moon's orbit quite round backward, in 18$\frac{2}{3}$ years.

Above the laſt-mentioned wheel of 59 teeth, and on the ſame axis with it, is a wheel of 55, turning a wheel of 62 teeth below the Earth; and this wheel of 62 moves the Moon's apogee plate quite round forward, in 8 years 312 days. And from this plate a wire riſes, and points out the place and motion of the apogee in the Moon's ecliptic.

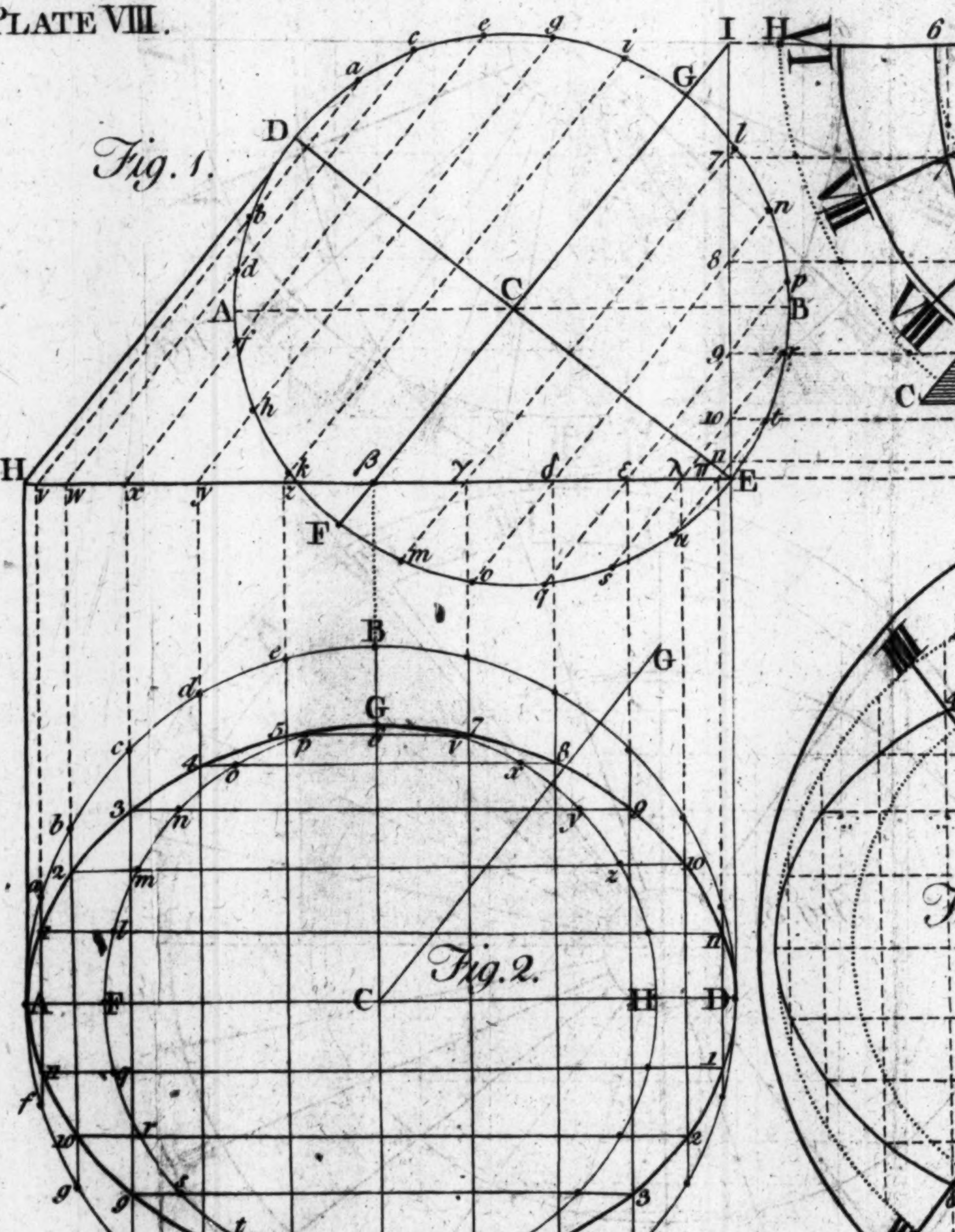
[*By comparing this deſcription with that of the Orrery in the 8th plate of my Aſtronomy* (ſee § 399.) *it will be very eaſily underſtood: particularly thoſe parts which ſhew the parallelism of the Earth's axis, the motion of the Moon's nodes, and apogee.*]

As the Moon goes round the Earth, ſhe comes to her mean changes, nodes, and apogee, in the proper times; and, at all intermediate times, her diſtance from her apogee and nodes are ſhewn in her ecliptic, orbit, and apogee plate; on which laſt her mean anomaly and elliptic equation are ſhewn: by which means her true place in the ecliptic, and her latitude, may be very nearly found for any given time.

The days of the months, throughout the year, are laid down in a diagonal manner, in a ſpiral line of four revolutions, marked 0, 1, 2, 3, for leap year, and the firſt, ſecond, and third years after. The annual index, in theſe ſpirals, being at the given day of any month, for either of theſe years, all the other motions and phenomena will be right for that day: and, by means of theſe diagonals, the lunation is brought ſtill nearer the truth than as above ſpecified.

Within this ſet of ſpirals are tables, which ſhew the places of the Sun, Moon,

PLATE VIII.



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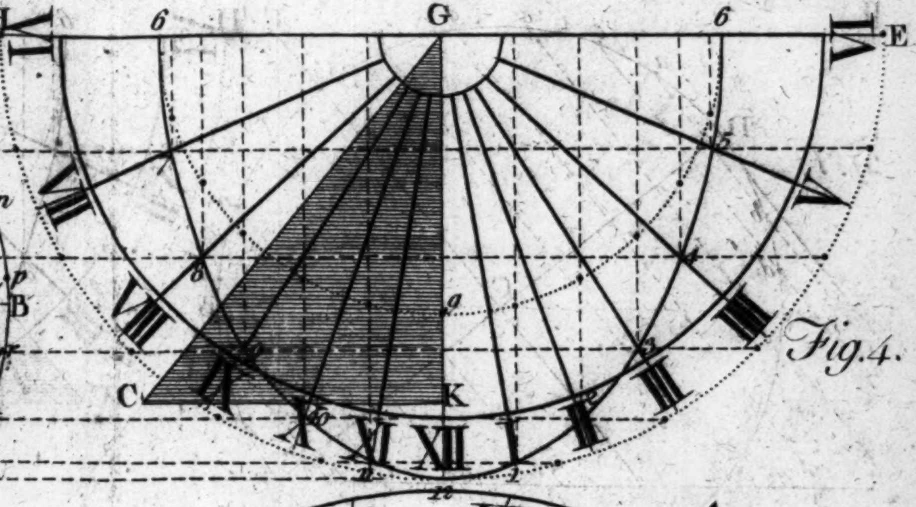


Fig.4.

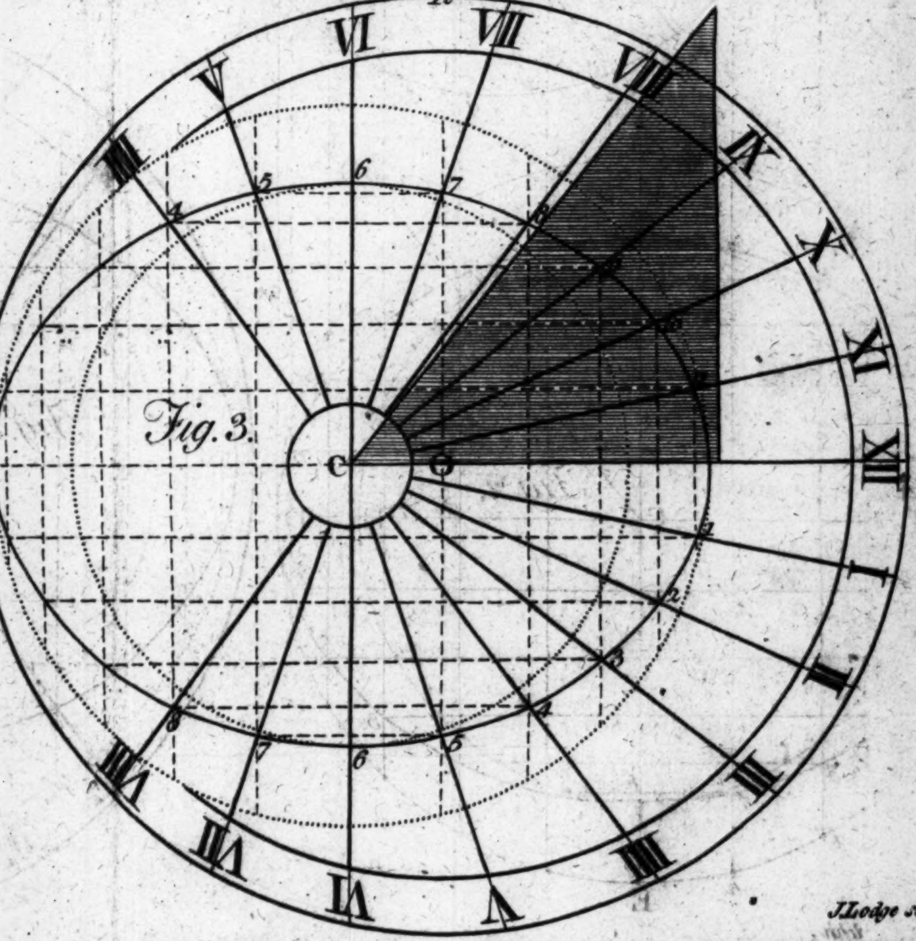


Fig.3.

J.Lodge sc.

Moon, Aſcending Node, and Apogee, for the noon of the firſt day of January, in any year within the limits of 6000 years both before or after the Chriſtian æra. And, by means of theſe tables, the Orrery may, in leſs than two minutes of time, be rectified for the beginning of any of theſe years; and then, all the motions, not only for that year, will be right, but alſo for 334 years afterward, without needing any rectification.

A New Geometrical Method of conſtructing Sun-Dials.

Draw at pleaſure the horizontal line *ACB* (Plate VIII. Fig. 1.) and on the point *C*, as a center, deſcribe the circle *DGEF*. Draw the diameter *DCE*, ſo as to make an angle *(DCA)* with *ACB* equal to the co-latitude of the place for which the dial is to ſerve; and draw *FCG* at right angles (or perpendipendicular) to *D C E*: then *A C B* ſhall repreſent the horizon of the place, *D C E* the equinoctial, *E C G* the axis of the world and ſtile of the dial, *G* the north pole, *F* the ſouth pole, and the arc *B G* the elevation of the pole above the north point of the horizon; which elevation is equal to the latitude of the place.

From the point *E* draw the right line *E H* parallel to the horizon *ACB*, and from the point *D* draw *D H* parallel to *CF*. So *E H* ſhall be equal to the longeſt diameter of an ellipſis (Fig. 2.) and *D E* equal to the ſhorteſt diameter thereof. Divide the circle *D G E F* into 24 equal parts, beginning at *D*; and connect the diviſion-points which are equidiſtant from *D* by the ſtraight lines *a b*, *c d*, *e f*, &c. continuing theſe lines down to the points *v*, *w*, *x*, *y*, *z*, in that part of the line *H E* that falls without the circle.

From the point β, where *G C F* interſects *H E*, draw β *B C E* (Fig. 2.)

perpendicular to *Hβ E* (Fig. 1.) and draw *ACD* in Fig. 2. parallel to *HβE* in Fig. 1. So, in Fig. 2. *B C E* and *ACD* ſhall croſs each other at right angles in the point *C.*—On this point, as a center, with the length *β H* or *β E* in Fig. 1. as a radius, deſcribe the circle *ABDE* in Fig. 2. and divide it into 24 equal parts, beginning at *A*, and connect the diviſion-points, which are equidiſtant from *A*, by the ſtraight lines *a f*, *b g*, *c h*, *d i*, *e k*, &c. Then, from Fig. 1. take *CD* in your compaſſes, as a radius; and, with that extent, on *C*, as a center in Fig. 2. deſcribe the circle *FGHI*, and divide it into 24 equal parts, beginning at *G*. Through theſe diviſion-points *p*, *v*, *o*, *x*, *n*, *y*, &c. which are equidiſtant from *G*, draw the right lines 5 7, 4 8, 3 9, 2 10, &c. meeting the lines within the former circle at the points 5, 7, 4, 8, 3, 9, 2, 10, 1, 11, on the ſide *AGD*; and at 7, 5, 8, 4, 9, 3, 10, 2, 11, 1, on the ſide *AID*. Then, through theſe points of meeting, draw by a ſteady hand the ellipſis

A 1 2 3 4 5 6 7 8, &c. whoſe longeſt diameter *A D* is equal to *H β E* in Fig. 1. and its ſhorteſt diameter *G I* equal to *D C E* in the ſame figure, as above mentioned.

This done (which may be much ſooner done than deſcribed) lay the edge of a ſtraight ruler to the center *C* in Fig. 2. and to the above-mentioned diviſion-points 5, 7, 4, 8, &c. in the ellipſis, and draw ſtraight lines from *C* through theſe points, as in Fig. 3.; and they will be the true hour-lines on a horizontal dial.

Laſtly, from the center *C*, in Fig. 2. draw the ſtraight line *C G* parallel to *CG* in Fig. 1. for the axis of the ſtile, or edge thereof that caſts a ſhadow on the time of the day; and *CG* ſhall be parallel to the axis of the world when the dial is truly ſet, as the like edge of the ſtile of every dial muſt be.

[*N. B.* Straight lines, parallel to *BCE* in Fig. 2. being drawn through the ellipſis from the points (Fig. 1.) *v, w,*

v, w, x, y, z, β, γ, δ, ε, λ, π, will cut the ellipſis in the points 1 11, 2 10, 3 9, &c. through which the hour-lines on the horizontal dial muſt be drawn from the center C.]

The point C (Fig. 3.) from which the hour-lines are drawn, ſhould not coincide with the center O of the dial-plate, but be taken at ſome ſmall diſtance from it, toward the left-hand from XII in the meridian line, in order to enlarge the ſpaces between the hours near mid-day, as the angular diſtances between them are leſs than thoſe about VI in the morning or afternoon.

From the point E in Fig. 1. draw EI perpendicular to EH, till it meets the axis FCG (produced beyond G) in I. Then, as $H\beta E$ repreſents a horizontal plane, ſo EI will repreſent a vertical one, facing the ſouth; and ſerve for ſhewing how to draw the hour-lines on an erect direct ſouth dial, as Fig. 4.

From the point *I* in Fig. 1. draw *IHGE* perpendicular to *EI*: and make *GH* in Fig. 4. equal to *EI* in Fig. 1. With this extent, as a radius, ſet one foot of the compaſſes in *G* (Fig. 4.) and with the other foot deſcribe the ſemicircle *H* 12 *E*, and divide it into 12 equal parts. Through theſe points of diviſion, which are equidiſtant from 12, draw the ſtraight lines 11 1, 10 2, 9 3, 8 4, and 7 5. Make *G* 6 in Fig. 4. equal to *CD* in Fig. 1. and with that extent, as a radius, ſet one foot of the compaſſes in *G* (Fig. 4.) and with the other foot deſcribe the ſemicircle 6 *g* 6. Then divide this ſemicircle into 12 equal parts, and thro' theſe points of diviſion draw ſtraight lines parallel to *Gg*, meeting the former ſtraight lines (drawn parallel to *HE*) in the points 7 5, 8 4, 9 3, &c. and draw the ſemi-ellipſis 6 7 8 9 10 11 12 1 2 3 4 5 6 through theſe points. So the longeſt diameter *G* 12 of this ſemi-ellipſis ſhall be equal to *IE* in

Fig. 1. and its ſhorteſt diameter 6 *G* 6 equal to *DCE*. This done, lay the edge of a ruler to the center *G* (Fig. 4.) and ſucceſſively to all the diviſion-points 7, 8, 9, &c. in the ſame ellipſis, and draw ſtraight lines from *G* thro' theſe points, as in the Figure, and they will be the true hour lines on the erect direct ſouth dial; to which ſet the hours, as in Fig. 4. and draw *G C* therein parallel to *G C* in Fig. 1. for the hypothenuſe or axis of the ſtile that caſts a ſhadow on the time of the day; and the dial will be finiſhed when the ſtile *G C K* is erected upon the 12 o'clock line *G g K* 12. And when the dial is truly ſet, the edge *G C* of the ſtile will be parallel to the axis of the world.

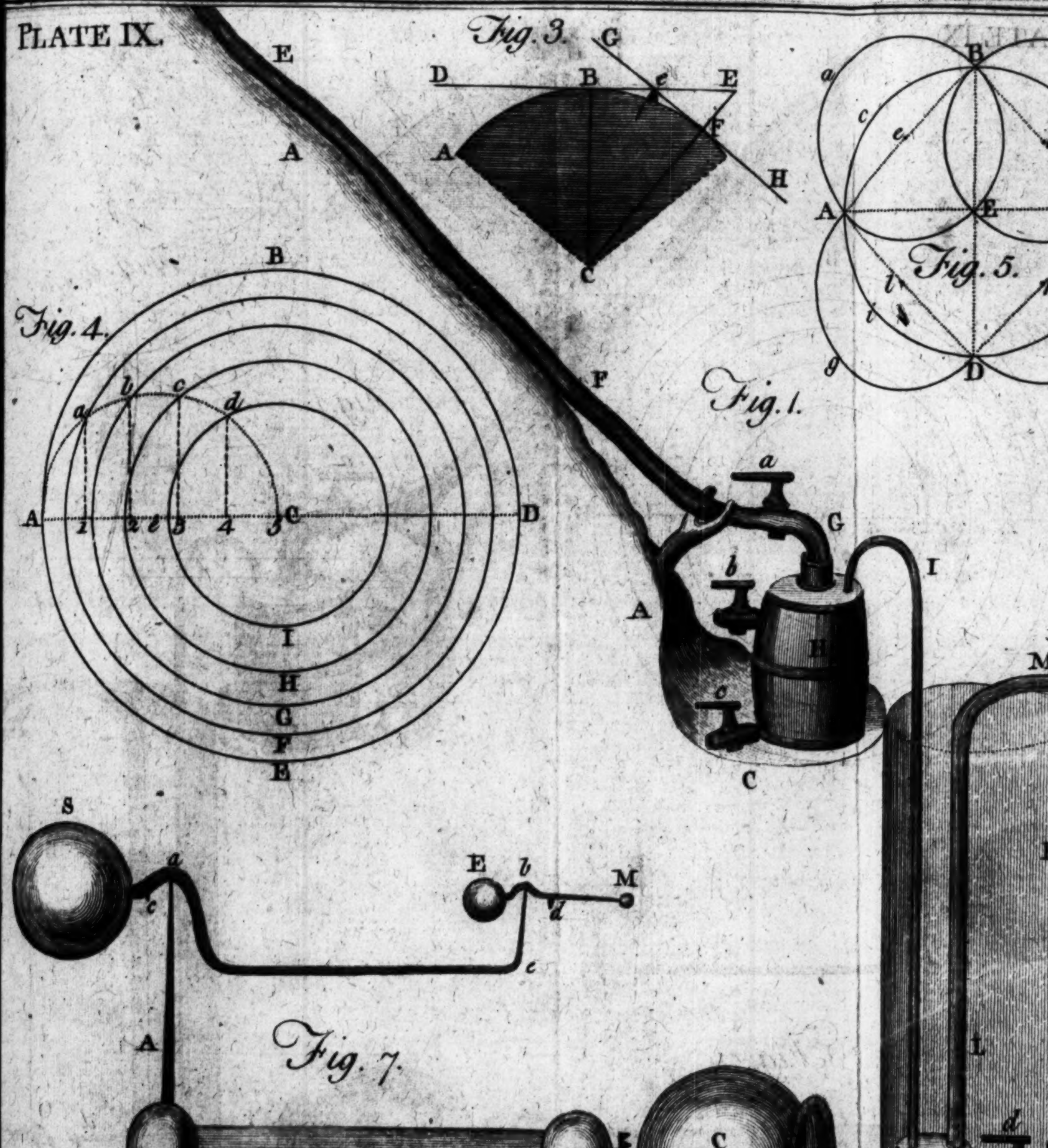
[*N. B.* Straight lines drawn parallel to *HGE*, thro' the ſemi-ellipſis 6 12 6, from the points (Fig. 1.) 7, 8, 9, 10, 11, in the perpendicular *E I*, where the lines *l m*, *n o*, *p q*, *r s*, and *t u*, cut this perpendicular, will cut the ſemi-ellipſis in Fig. 4. in thoſe points through which the hour-lines muſt be drawn from the center *G.*]

And thus by means of a circle divided into 24 equal parts, as in Fig. 1. a horizontal dial or erect direct ſouth dial may be made for any latitude. The method may perhaps appear tedious, on account of the number of words in the deſcription: but I will venture to ſay, that whoever puts it in practice, will find it ſhort, eaſy, and pleaſant.

Deſcription of the Hungarian Machine for raiſing Water from Mines.

In Fig. 1. of Plate IX. *A A* is the ſide of a hill, cloſe by the brink of the ſhaft or mine-pit *B B*, which is 104 feet deep below the ſurface of the ground *C* at the foot of the hill. In this hill is

PLATE IX.



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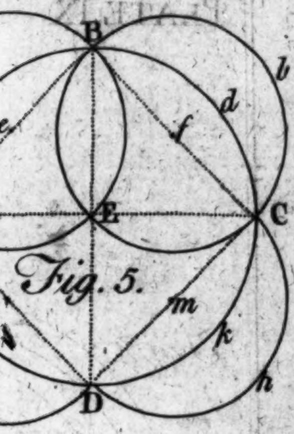
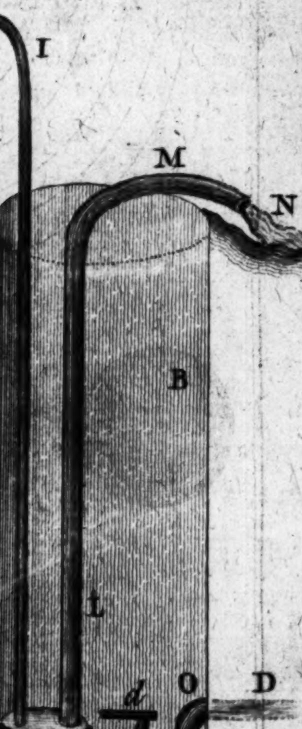


Fig. 5.



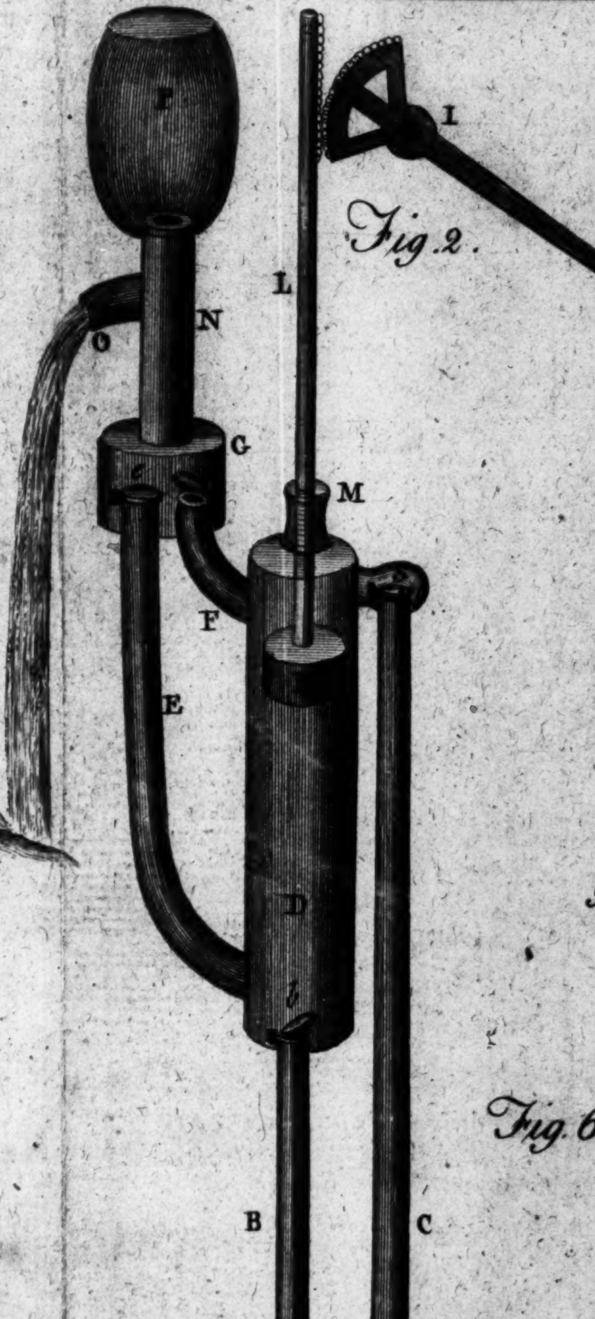


Fig. 2.

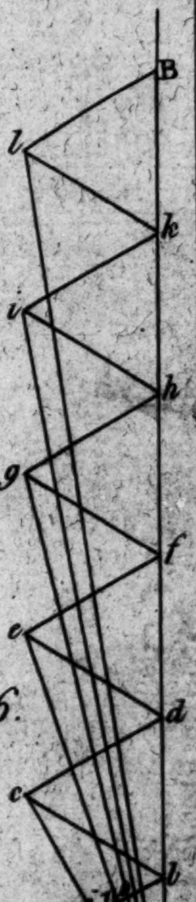


Fig. 6.

J. Lodge sculp.

is a large ſpring of water, 143 feet above the ſurface of the ground at *C* (taken in perpendicular meaſure) and the ſpring affords much more water than what the ſpring *D*, under ground, lets in to the mine.

A pipe *E F G* lets the water down from the ſpring in the hill, into a cloſe air-tight veſſel *H* that ſtands at the foot of the hill, that contains 57½ cubic feet, or 43 gallons in wine meaſure. In this pipe is a cock *a*, which being opened or ſhut, lets the water of the ſpring run into *H*, or ſtops it, as occaſion requires: and in *H* are two cocks *b* and *c*, the uppermoſt of which is for letting air into *H*, and the lowermoſt for letting the water out of it.

A ſmall pipe *I* goes from the veſſel *H* on the ſurface of the ground to a veſſel *K* in the bottom of the mine, and terminates in the top thereof. This veſſel is air-tight, and contains 27½ cubic feet, or 205⅓ gallons in wine meaſure, which is forced up the aſcending pipe *L M*, and runs off to waſte, at *N*, above ground. The lower end of this pipe goes down ſo far into the veſſel *K*, as almoſt to touch its bottom.

From this veſſel, a pipe *O* goes to the ſpring *D* under ground, which lets water into the mine, and would overflow it if the water was not forced up or raiſed from the mine, through the pipe *L M*. The pipe *O* lets this water into the veſſel *K* when the cock *d* is turned open, and keeps back the water when the cock is ſhut.

The Operation is as follows:

The cock *b* being open, and the cocks *a* and *c* ſhut, and no water in the veſſel *K*, open the cock *d* to let the veſſel *K* fill with water from the ſpring in the mine. As this veſſel fills, the water will drive the air out of it, up through the ſmall pipe *I* into the veſſel *H*, and all *that* air will go out of the veſſel *H* by the open cock *b*, and then *H* will remain, as it was before, full of air in the ſame ſtate of denſity as the common air is on the outſide of *H*. When *K* is full of water, ſhut the cocks *b* and *d*, and open the cock *a* to let water run down from the ſpring in the hill, by the pipe *E F*, into the veſſel *H*. As the water riſes in that veſſel, the air will thereby be driven out of it, down through the pipe *I*, into the veſſel *K*: and as this air is compreſſed by the weight of the running water in the pipe *E F*, the compreſſed air will force all the water out of the veſſel *K*, up through the pipe *L M*, from which it will run off at *N* on the ſurface of the ground; and then the compreſſed air will ruſh out, after the water.

When the veſſel *K* in the mine is emptied of water, and the air is heard to begin to ruſh out, ſhut the cock *a* to ſtop the water from the ſpring, and open the cocks *b* and *c*: then the water that came from the ſpring will run out of the veſſel *H* by the cock *c*, and air will go in by the cock *b*: at the ſame time, open the cock *d* in the mine, to let the veſſel *K* fill with water from *D* the ſpring in the mine; and as *H* empties above-ground, *K* will fill below it; and the air that remained in *K* will (by the riſing of the water in it) be driven back into the veſſel *H* through the pipe *I*.

When *H* is empty of water, and *K* full, ſhut the cocks *b*, *c*, and *d*, and open the cock *a*: then *H* will fill with water from the ſpring in the hill; and this water, as it riſes in *H*, will force the air out of *H*, down the pipe *I*, upon the water in *K*; and the force of the compreſſed air will drive all the water out of *K*, up the pipe *L M*, from which it will run off at *N*, as before.

And thus, wherever there is a ſpring in a hill, near a mine, that affords more water than what flows into the mine from a ſpring under ground; and the perpendicular height of the ſpring in the hill is greater than the depth of the mine; water may thus be raiſed from the mine, in a moſt ſimple and eaſy manner, by an engine in which there are neither pumps, piſtons, nor valves: and ſuch an engine will not be liable to be out of order, nor need repairs in many years. As this engine is but very little known in Britain, I have made a working model of it, which I always ſhew in my courſe of lectures.

But as there are very few mines that have hills near them with high ſprings, water cannot then be raiſed from them in this manner; and therefore Mr. Blakey propoſed another method, which was, to make *H* an air-veſſel, with a pipe going from it to another veſſel in which is water, kept boiling by a fire under it, and this veſſel to have a cock to let out the ſteam occaſionally that riſes from the ſurface of the boiling water. When the cock is ſhut, the ſteam will go off from the boiler into the air-veſſel *H*, and drive the air out of it, down through the pipe *I* into the veſſel *K* in the mine: and the force of the air compreſſed by the elaſticity of the ſteam, will raiſe the water from *K*, up through the pipe *L M*, till *K* be emptied of water. Then the cock in the boiler is to be turned open, to let out the ſteam, and the cock *d* to be opened to let the veſſel *K* fill from the ſpring in the mine: and when it was full, both theſe cocks are to be ſhut, and the operation will go on as before.

That Blakey's ſcheme would do, the Hungarian machine puts beyond all doubt.—In both of them the veſſels muſt be made very ſtrong, becauſe every part of each veſſel, equal in ſurface to the bore of the aſcending pipe *L M*, will ſuſtain an outward preſſure equal to the whole weight of water in that pipe.—It will not anſwer for ſuch depths as the common fire-engine will, nor will it raiſe ſo much water; but it may be built for leſs than a third part of the expence, and would anſwer very well where the depth is not above 100 feet.

Deſcription of a pump, invented by M. De-la-Hire, *which raiſes Water equally quick by the Deſcent as by the Aſcent of the Piſton in the Pump-Barrel.*

In Fig. 2. of Plate IX. *A A* is the well, in which the lower ends of the pipes *B* and *C* are placed. *D* is the pump-barrel, into the lowermoſt end of which the top of the open pipe *B* is ſoldered, and in the uppermoſt end the hollow piece *S* is ſoldered, which opens into the barrel, and the top of the pipe *C* is ſoldered into that piece. Each of theſe pipes has a valve on its top, and ſo have the crooked pipes *E* and *F*, whoſe lower ends are open into the pump-barrel, and their upper ends into the box *G*.

H is the pump-handle, its center of motion is at *I*; and as it is moved up and down, it moves the ſolid plunger *K* up and down in the barrel, by the ſtraight rod or ſpear *L*, which moves air-tight in a long collar of leathers in the neck *M*; and the plunger never goes higher than *K*, nor lower than *D*; ſo that from *K* to *D* is the length of the ſtroke.

As the plunger riſes from *D* to *K*, the atmoſphere (preſſing on the ſurface of the water *A A* in the well) forces the water up the pipe *B*, through the valve *b*, and fills the pump-barrel with water up to the plunger: and during this time, the valves *c* and *S* lie cloſe and air-tight on the tops of the pipes *E* and *C*.

When the plunger is up to its greateſt height at *K*, it ſtops there for an inſtant; and in that inſtant the valve *b* falls, and ſtops the pipe *B* at top. Then, as the plunger goes down, it cannot force the water between *K* and *D* back through the cloſe valve *b*, but forces all *that* water up through the crooked pipe *E* through the valve *e*, which then opens upward by the force of the water; and this water, after having filled the box *G*, riſes into the pipe *N*, and runs off by the ſpout at O.

During the deſcent of the plunger *K*, the valve *f* falls down, and covers the top of the crooked pipe *F*; and the preſſure of the atmoſphere on the well *AA* forces water up the pipe *C*, through the valve *S*, which then opens upward by the force of the aſcending water; and this water runs from *S* into the pump-barrel, and fills all the ſpace in it above the plunger.

When the plunger is down to its loweſt deſcent at *D*, and ſtops there for an inſtant, in that inſtant the valve *S* falls down, and ſhuts the top of the pipe *C*: and then, as the plunger is raiſed, it cannot force the water above it back through the valve *S*, but drives all that water up the crooked pipe *F*, through the valve *f*, which opens upward by the force of the aſcending water; which water, after filling the box *G*, is forced up from thence into the pipe *N*, and runs off by the ſpout at *O*.

And thus, as the plunger deſcends, it forces the water below it up the pipe *E*; and as it aſcends, it forces the water above it up the pipe *F*; the preſſure of the atmoſphere filling the pump-barrel below the plunger thro' the pipe *B* while the plunger aſcends, and filling the barrel with water above the plunger, through the pipe *C*, as the plunger goes down.

And thus, there is as much water forced up the pipe *N* to the ſpout *O* by the deſcent of the plunger, as by its aſcent; and, in each caſe, as much water diſcharged at *O* as fills that part of the pump-barrel as the plunger moves up and down in.

On the top of the pipe *O* is a cloſe air-veſſel *P*. When the water is forced up above the ſpout *O*, it compreſſes the air in the veſſel *P*; and this air, by the force of its ſpring acting on the water, cauſes the water to run off by the ſpout *O* in a conſtant (and very nearly) equal ſtream.

Whatever the height of the ſpout *O* be above the ſurface of the well, the top *S* of the pipe *C* muſt not be 32 feet above that ſurface; becauſe, if that pipe could be entirely exhauſted of air, the preſſure of the atmoſphere in the well would not force the water up the pipe to a greater height than 32 feet. And if *S* be within 24 feet of the ſurface of the well, the pump will be ſo much the better.

As the collar of leathers within the neck *M* are apt to dry and ſhrink when the pump is not uſed, and conſequently to let air get into the pump-barrel, which would ſtop the operation of the atmoſphere in the pipe *C*, I think collars of old hats might be uſed inſtead of leathers, as they would not be liable to that inconvenience.

It matters little what the ſize of the pipe *N* be, through which the water is forced up to the ſpout: but a great deal depends on the ſize of the pump-barrel; and, according to the height of the ſpout *O* above the ſurface of the well, the diameter of the bore of the barrel ſhould be as follows.

For 10 feet high, the bore ſhould be 6.9 inches; for 15 feet, 5.6; for 20 feet, 4.9; for 25 feet 4.4; for 30 feet, 4.0; for 35 feet, 3.7 inches; for 40 feet, 3.5; for 45 feet, 3.3; for 50 feet, 3.1; for 55 feet, 2.9; for 60 feet, 2.8; for 65 feet, 2.7; for 70 feet, 2.6; for 75 feet, 2.5; for 80 feet, 2.5 will do; for 85 feet, 2.4; for 90 feet 2.3; for 95 feet, 2.2; and for 100 feet, the diameter of the bore ſhould not exceed 2.1 or 2.2 inches at moſt.—If theſe proportions are attended to, a man of common ſtrength may raiſe water 100 feet high by one pump as eaſily as he could raiſe it 10 feet high by another.

In this pump, the pipes *B* and *C* ſeem to be rather too ſmall, which will cauſe the water riſing in them to have a great deal of friction, from the quickneſs of its motion: and whoever makes ſuch a pump, will find it very difficult to make the leathers in the neck *M* water-tight, ſo as that no water ſhall be forced out that way when the piſton is drawn up.

The Height of the apparent Level above the true.

In Fig. 3. of Plate IX. let *ABF* be part of the Earth's ſpherical ſurface, *C* the Earth's center, *BC* or *FC* its ſemidiameter, *DBE* a tangent to the Earth's ſurface at *B*, drawn perpendicular to *BC*; and *GFH* a tangent to the Earth's ſurface at *F*, drawn perpendicular to *FC*. The line *DBE* is a true level at *B*; but being carried on ſtraight toward *D* or *E*, it riſes above the Earth's ſurface: and although it ſeems to be level as ſeen from *B*, it is above the true level at *F*, by the whole height *FE*; for, at the point *F*, the tangent *GFH* is the true level.

At the diſtance of a geographical mile, or 6094 feet, from the point *B*, the line *BD* or *BE* will be 10.637 inches above the globular ſurface of the Earth: at two ſuch miles from *B*, the ſame line will be four times as high (or 3 feet 6.548 inches) above the Earth's ſurface: at three miles diſtance, nine times as high (or 7 feet 11.733 inches) above the Earth's ſurface; and ſo on, always increaſing in height according to the ſquare of the diſtance: for, if *F* be twice the diſtance of *f* from *B*, *FE* muſt be four times as high as *fe*, if both their tops touch the right line *BeE*.

At the diſtance of an Engliſh mile, or 5280 feet, the apparent level is 7.90 inches higher than the true; at two miles diſtance, it is four times as high, or 2 feet 7.60 inches above the true;

at three miles diſtance, nine times as high; and ſo on, increaſing in proportion to the ſquare of the diſtance.

At the diſtance of a degree or 60 geographical miles, which are equal to $69\frac{1}{4}$ Engliſh miles, the height of the apparent level above the true is 3191 feet and $\frac{9}{10}$ parts of an inch. And therefore, a hill, whoſe top was ſo far above the level of the ſea, would be juſt ſeen at top by an eye at the ſurface of the ſea, and $69\frac{1}{4}$ Engliſh miles from the hill.

Of the Velocities acquired by falling Bodies, and the Spaces they fall through in given Times.

In ſucceſſive equal parts of time, as 1, 1, 1, 1, &c. the ſpaces thro' which a body falls are as 1, 3, 5, 7, &c. and the acquired velocities are as 1, 2, 3, 4, &c. continually: ſo that the velocities are as the times, and the ſpaces are as the ſquares of the times in falling.

Thus, in the firſt ſecond of time (from the inſtant of beginning to fall) the body will fall through 16 feet; in the next ſecond it will fall through three times 16, or 48 feet, which added to the former 16 makes 64 feet, the whole ſpace fallen through in 2 ſeconds of time: in the third ſecond of time, the body falls 5 times 16, or 80 feet; which, added to the above 64, makes 144 feet, the whole ſpace fallen through in 3 ſeconds: in the fourth ſecond it falls 7 times 16, or 112 feet; which, added to the above 144 feet, makes 256, the whole ſpace fallen through in 4 ſeconds: and ſo on continually, increaſing as the odd numbers 1, 3, 5, 7, 9, 11, in 1, 2, 3, 4, 5, 6 ſeconds of time.

Whatever velocity the body acquires at the end of the firſt ſecond, it will acquire twice as much at the end of the next, three times as much at the end of the third, four times as much at the end of the fourth ſecond, and ſo on continually.

In the following Table, the numbers under T denote the ſeconds of time, from 1 to 60, in which the body continues to fall: the numbers under S denote the ſpaces, in feet, through which the body falls in any ſecond from 1 to 60: and the numbers under N denote the whole number of feet the body falls through, at the end of any number of ſeconds from 1 to 60. Thus, between the end of the 59th and 60th ſecond, the body falls 119 times 16 feet; and at the end of the 60th ſecond it has fallen through 57600 feet.

In a quarter of a ſecond from the inſtant of beginning to fall, a body would fall one foot: at the end of half that ſecond it will have fallen 4 feet: at the end of three quarters of that ſecond it will have fallen through 9 feet: and at the end of that whole ſecond, through 16.

The whole ſpaces fallen through being as the ſquares of the times in which the body falls, Qu. How many feet would it fall through in an hour?

In 60 ſeconds the ſpace is 57600 feet, and the ſquare of 60 is 3600. But 57600 multiplied by 3600 is 207360000, the number of feet the body would fall through in an hour, in a free or unreſiſting ſpace: and this number being divided by 5280, the number of feet in an Engliſh mile, quotes 39272.7 for the number of miles.

TABLE *of* FALLING BODIES.

T.	S.	N.	T.	S.	N.	T.	S.	N.	T.	S.	N.
1	1	Feet 16	16	31	4096	31	61	15376	46	91	33856
2	3	64	17	33	4624	32	63	16384	47	93	35344
3	5	144	18	35	5184	33	65	17424	48	95	36864
4	7	256	19	37	5776	34	67	18496	49	97	38416
5	9	400	20	39	6400	35	69	19600	50	99	40000
6	11	576	21	41	7056	36	71	20736	51	101	41616
7	13	784	22	43	7744	37	73	21904	52	103	43264
8	15	1024	23	45	8464	38	75	23104	53	105	44944
9	17	1296	24	47	9216	39	77	24336	54	107	46656
10	19	1600	25	49	10000	40	79	25600	55	109	48400
11	21	1936	26	51	10816	41	81	26896	56	111	50176
12	23	2304	27	53	11664	42	83	28224	57	113	51984
13	25	2704	28	55	12544	43	85	29584	58	115	53824
14	27	3136	29	57	13456	44	87	30976	59	117	55696
15	29	3600	30	59	14400	45	89	32400	60	119	57600

A

A TABLE *ſhewing how much the Mercury would ſink in a Barometer at given Heights above the Earth's plane Surface; and conſequently, how the Heights of Hills may be found thereby.*

At the height of	Merc. ſinks		At the height of	Merc. ſinks		At the height of	Merc. ſinks		At the height of	Merc. ſinks		At the height of	Merc. ſinks	
Feet.	Inches.	100 parts.	Feet.	Inches.	100 parts.	Feet.	Inches.	100 parts.	Feet	Inches.	100 parts.	Feet.	Inches.	100 parts.
100	0	.11	3700	3	.83	7300	7	.05	10900	9	.87	14500	12	.37
200	0	.22	3800	3	.92	7400	7	.13	11000	9	.94	14600	12	.44
300	0	.33	3900	4	.02	7500	7	.22	11100	10	.01	14700	12	.50
400	0	.44	4000	4	.12	7600	7	.30	11200	10	.08	14800	12	.57
500	0	.54	410	4	.21	7700	7	.38	11300	10	.16	14900	12	.63
600	0	.65	4200	4	.30	7800	7	.46	11400	10	.23	15000	12	.70
700	0	.76	4300	4	.39	7900	7	.55	11500	10	.30	15100	12	.76
800	0	.87	4400	4	.49	8000	7	.63	11600	10	.37	15200	12	.83
900	0	.98	4500	4	.58	8100	7	.71	11700	10	.44	15300	12	.89
1000	1	.09	4600	4	.67	8200	7	.79	11800	10	.52	15400	12	.96
1100	1	.19	4700	4	.77	8300	7	.87	11900	10	.59	15500	13	.02
1200	1	.30	4800	4	.86	8400	7	.95	12000	10	.66	15600	13	.09
1300	1	.40	4900	4	.95	8500	8	.03	12100	10	.73	15700	13	.15
1400	1	.50	5000	5	.04	8600	8	.11	12200	10	.80	15800	13	.21
1500	1	.61	5100	5	.13	8700	8	.19	12300	10	.87	15900	13	.28
1600	1	.72	5200	5	.22	8800	8	.27	12400	10	.94	16000	13	.34
1700	1	.82	5300	5	.31	8900	8	.35	12500	11	.01	16100	13	.40
1800	1	.93	5400	5	.40	9000	8	.43	12600	11	.08	16200	13	.47
1900	2	.03	5500	5	.49	9100	8	.51	12700	11	.15	16300	13	.53
2000	2	.14	5600	5	.58	9200	8	.58	12800	11	.22	16400	13	.59
2100	2	.24	5700	5	.67	9300	8	.66	12900	11	.29	16500	13	.65
2200	2	.34	5800	5	.76	9400	8	.74	13000	11	.36	16600	13	.71
2300	2	.44	5900	5	.85	9500	8	.82	13100	11	.43	16700	13	.78
2400	2	.54	6000	5	.94	9600	8	.89	13200	11	.50	16800	13	.84
2500	2	.65	6100	6	.02	9700	8	.97	13300	11	.56	16900	13	.90
2600	2	.75	6200	6	.11	9800	9	.05	13400	11	.63	17000	13	.96
2700	2	.85	6300	6	.20	9900	9	.12	13500	11	.70	17100	14	.02
2800	2	.95	6400	6	.28	10000	9	.20	13600	11	.77	17200	14	.08
2900	3	.05	6500	6	.37	10100	9	.27	13700	11	.84	17300	14	.15
3000	3	.15	6600	6	.45	10200	9	.34	13800	11	.90	17400	14	.21
3100	3	.25	6700	6	.54	10300	9	.42	13900	11	.97	17500	14	.27
3200	3	.34	6800	6	.63	10400	9	.50	14000	12	.04	17600	14	.33
3300	3	.44	6900	6	.71	10500	9	.57	14100	12	.11	17700	14	.39
3400	3	.54	7000	6	.80	10600	9	.64	14200	12	.17	17800	14	.45
3500	3	.63	7100	6	.88	10700	9	.72	14300	12	.24	17900	14	.51
3600	3	.73	7200	6	.97	10800	9	.79	14400	12	.30	18000	14	.57

By this Table, and a common barometer, the height of any hill may be found, if its height, taken in perpendicular meaſure, be not much above half a mile.—Thus, if the mercury be 2 inches and 95 hundred parts of an inch lower in the tube at the top of the hill, than what it was obſerved to be at the bottom, the perpendicular height of the hill is 2800 feet, which is 160 feet more than half a mile.

But as there are many hills much higher than 2800 feet, and the common barometer-ſcale is only 3 inches long, let a ſcale 14 inches long, divided into inches, and hundred parts of an inch by diagonal lines, be applied to the tube, and have a ſliding index acroſs it in the common way; and this, I apprehend, will do for the higheſt mountain on the earth.—For, ſuppoſing the quickſilver was obſerved to be 13 inches and 21 hundred parts of an inch lower in the tube when at the top of the hill than it was when at the foot: againſt 13.21 inches in the Table is 15800 feet for the height of the hill, which wants only 40 feet of being 3 miles high.

To divide the Area of a given Circle into any required Number of equal Parts, by concentric Circles.

In Fig. 4. of Plate IX. let *ABDE* be a circle, whoſe area is required to be divided into 5 equal parts by concentric circles, as *FGHI.*

Divide the ſemidiameter *AC* into 5 equal parts, as *A* 1, 1 2, 2 3, 3 4, 4 5; and on the middle point *e* as a center, with the radius *e A*, deſcribe the ſemicircle *AabcdC*. From the points of equal diviſion at 1, 2, 3, and 4, and perpendicular to *AC*, raiſe the perpendiculars 1 *a*, 2 *b*, 3 *c*, 4 *d*, till they meet the ſemicircle in the points *a*, *b*, *c*, and *d*: through which points, draw the concentric circles *F*, *G*, *H*, *I*, and the thing will be done.

Suppoſing that five blackſmiths ſhould agree to buy a grinding-ſtone among them, each paying an equal ſhare of the price, and that each man ſhould therefore have the uſe of the ſtone, to wear off a fifth part of it, till it came to the laſt man, who was to wear it out: the firſt man ſhould wear the ſtone from *E* to *F*, the ſecond from *F* to *G*, the third from *G* to *H*, the fourth from *H* to *I*, and the fifth from *I* to the center or axle *C*.

By this eaſy method, which I learnt of Mr. Hutton, teacher of the Mathematics at Newcaſtle, the area of any circle may be divided by concentric circles into any required number of equal parts. For, into whatever number of equal parts the radius *AC* be divided, the area of the circle will be divided into the like number of parts, all equal among themſelves.

To

To make two equal Circles, whoſe Areas taken together, ſhall be equal to the Area of a given Circle: or four equal Creſcents, the Sum of whoſe Areas ſhall be equal to the Area of a given Square.

In Fig. 5. of Plate IX. let *c d k i* be the given circle. In this circle deſcribe the ſquare *e f m l*; and, on the middle points of any two of its ſides, as at *e* and *f* as centers, deſcribe the two circles *A a B E A* and *B b C E B*: the areas of theſe two circles, taken together, ſhall be equal to the area of the given circle *c d k i*.

Draw the diagonal *A E C*, which will divide the ſquare into two triangles *A B C* and *A D C*, right angled at *B* and *D*. Now, as the ſides *A B* and *BC* of the triangle *ABC* are equal, and ſo are the ſides *A D* and *D C* of the triangle *A D C*, and the areas of circles being as the ſquares of their diameters,

meters, and the hypothenuſe *A C* being ſquared is equal to the ſquare of the ſides *AB* and *B C*, or *A D* and *DC*; the larger ſemicircle *AcBdC* is equal to the two leſſer ſemicircles *A a B e A* and *B d C f B*. Conſequently, if you ſubtract the two common portions *AcBeA* and *BdCfB*, the two remaining creſcents *AaBcA* and *BbCdB* will be equal to the two triangles *AEBA* and *BCEB*, which make one half of the ſquare *e f m l*: and therefore the ſum of the areas of all the four outward creſcents is equal to the area of the whole ſquare.

Of ſquaring the Circle.

Although there has not yet been any method found for doing this to mathematical exactneſs, yet, by means of the following numbers, it may be brought ſo very near the truth, as to be within a grain of ſand in a ſquare

mile,

mile, ſuppoſing 100 grains of ſand (placed in a ſtraight line, and touching one another) to be equal to the length of an inch; and conſequently 4014489600000 to cover a ſquare mile.

If the diameter of a circle be given, and the length of the ſide of a ſquare ſo nearly equal to the circle as to be true to 14 places of figures be required; ſay, As 1 is to the diameter of the given circle, ſo is 0.88622692545276 to the ſide of the ſquare required, in ſuch meaſures as the diameter of the circle was taken.

If the length of the ſide of a ſquare be given, and the diameter of a circle equal (as nearly as above-mentioned) to the ſquare be required; ſay, As 1 is to the ſide of the given ſquare, taken in any meaſure, as feet, inches, *&c.* ſo is 1.12837916709551 to the diameter of a circle (taken in the ſame kind of meaſures) whoſe area is equal to the area of the given ſquare.

In practice, it is ſufficient to take out the decimal parts to four places of figures; for, even by ſo ſmall a number, we come ſo near the truth as to be within a ten thouſandth part of the whole area of being perfectly true. And this is nearer than any one can pretend to delineate on paper.

Thus, ſuppoſing the diameter of a circle to be 12 inches, and that it is required to find the length of the ſide of a ſquare (or to make a ſquare) whoſe area ſhall be equal to the area of the circle; ſay, As 1 is to 12, ſo is .8862 to 10.6344 inches, the length of the ſide of the ſquare required.

Or, ſuppoſing the ſide of a ſquare to be 12 inches, and that it is required to find the diameter of a circle whoſe area ſhall be equal to the area of a ſquare; ſay, As 1 is to 12, ſo is 1.1284 (inſtead of 1.128379) to 13.5408 inches the diameter of the circle required.

Hence, as a ſquare veſſel, juſt one foot wide and one foot deep, would hold a cubic foot of water; a cylindrical veſſel 13.54 inches wide and one foot deep would hold a cubic foot of water too; at leaſt ſo near the truth, that no difference could be perceived.

The diameter of any circle is in proportion to its circumference, as 1 is to 3.1415926535897932384626434; or ~~as 1~~ is to 3.1416, near enough for practice.

Any circle is equal to a parallelogram, whoſe length is equal to half the circumference of the circle, and breadth equal to half the diameter. Therefore multiply half the circumference by half the diameter, and the product ſhall be equal to the area of the circle, in ſquare meaſure. The ſquare root of this area is the ſide of a ſquare equal to the circle.

To ſhew that an Angle may be continually diminiſhed, and yet never be reduced to nothing: and conſequently, that Matter is infinitely diviſible.

In Fig. 6. of Plate IX. let *AB* be a ſtraight line, produced to an infinite length beyond *B*, and ſtraight throughout. On this line let there be an infinite number of equilateral triangles placed, as *A a b*, *b c d*, *d e f*, *f g h*, &c. whoſe baſes *A b*, *b d*, *d f*, *f h*, &c. touch one another upon the right line *AB*: and let the ſide *a b* of the firſt triangle be of any given length, as ſuppoſe an inch, and each ſide of each triangle be of the ſame length with *a b*.

Then, from the point *A* draw the ſtraight line *A c* to the top of the ſecond triangle *b c d*; and *A c* ſhall cut *a b* in the middle point at *m*.

From the point *A* draw the ſtraight line *A e* to the top of the third triangle *d e f*; and *A e* ſhall cut *a b* at *n*, in two thirds of its length from *a*, leaving only one third remaining, from *n* to *b*.

From the point *A* draw the right line *A g* to the top of the fourth triangle *f g h*; and *A g* ſhall cut *a b* at *o*, in three fourth parts of its length from *a*; and conſequently leave one fourth of it remaining, from *o* to *b*.

Here it is plain that every line drawn from *A* to the top of the next triangle beyond that to which the laſt preceding line was drawn, will make a leſs angle with the line *A B* than the laſt preceding line did. But no right line drawn from the point *A* to the top of any triangle placed upon *A B*, even at an infinite diſtance from *A*, could ever coincide with the line *A B*, although every ſucceeding line will make a leſs angle with *A B* than the line laſt drawn before it did: and therefore the angle at *A* will be continually diminiſhing, but can never come to nothing. Conſequently, the whole line *a b* will never be exhauſted or quite cut off by any line drawn from *A* to the top of any triangle: and therefore, a part of it will ſtill remain between *a* and *b*; which proves that matter is infinitely diviſible.

A new Experiment in Electricity, ſhewing the Motions of the Sun, Earth, and Moon; by Edward King, *Eſq; of* Lincoln's-Inn.

The Sun and Earth go round the common center of gravity between them in a ſolar year, and the Earth and Moon go round the common center of gravity between *them* in a lunar month.—Theſe motions are repreſented by an electrical experiment, as follows:

In Fig. 7. of Plate IX. the ball *S* repreſents the Sun, *E* the Earth, and *M* the Moon, connected by bended wires *a c* and *b d*: *a* is the center of gravity between the Sun and Earth, and *b* is the center of gravity between the Earth and Moon. Theſe three balls, and their connecting wires, are hung and ſupported on the ſharp point of a wire *A*, which is ſtuck upright in the prime conductor *B* of the electrical machine; the Earth and Moon hanging upon the ſharp point of the wire *c a e*, in which wire is a pointed ſhort pin, ſticking out horizontally at *c*; and there is juſt ſuch another pin at *d*, ſticking out in the ſame manner, in the wire that connects the Earth and Moon.

When the globe *C* of the electrical machine is turned, the above-mentioned balls and wires are electrified: and the electrical fire, flying off horizontally from the points *c* and *d*, cauſe *S* and *E* to move round their common center of gravity *a*; and *E* and *M* to move round their common center of gravity *b*. And, as *E* and *M* are light when compared with *S* and *E*, there is much leſs friction on the point *b* than upon the point *a*; ſo that *E* and *M* will make many more revolutions about the point *b* than *S* and *E* make about the point *a*.—I had this experiment from my ingenious friend Mr. King; and have adjuſted the weights of the balls ſo, that *E* and *M* go twelve times round *b* in the time that *S* and *E* go only once round *a*.—It makes a good amuſing experiment in electricity; but is ſo far from proving that the motions of the planets in the heavens are owing to a like cauſe, that it plainly proves they are not. For the real Sun and Planets are not connected by wires or bars of metal; and conſequently there can be no ſuch metallic points as *a* and *b* between them. And without ſuch points, the electric fluid would never cauſe them to move: for, take away theſe points in the above-mentioned experiment, and the balls will continue at reſt, let them be ever ſo ſtrongly electrified.

TABLES

FOR

Calculating THE TRUE TIME of any NEW or FULL MOON, from the Creation of the World to A. D. 7800; near enough the Truth for any common Almanack.

ADVERTISEMENT.

In all the Britiſh Lunar Tables hitherto publiſhed, the time depending upon the Moon's annual equation is ſubtracted from the mean time of New and Full Moon when the Sun's anomaly is leſs than ſix ſigns, and added when greater: in the elliptic equation, the time depending on the Moon's anomaly is added to the mean time of New and Full Moon when her anomaly is leſs than ſix ſigns, and ſubtracted therefrom when her anomaly is greater. In the following Tables, I have made theſe equations always additive, which renders the calculations much eaſier. For this purpoſe, I have put down all the radical mean times of New and Full Moon 13 hours 59 minutes ſooner than in the former Tables; the greateſt annual equation being 4 hours 11 minutes, and the greateſt elliptic equation 9 hours 48 minutes; the ſum of both theſe (to the neareſt full minute) is 13 hours 59 minutes. The numbers under A *are the degrees of the Sun's mean anomaly, and thoſe under* B *the degrees of the Moon's—I was led to this, by obſerving that the late eminent* M. CLAIRAUT *at Paris has made all the equations additive in his Lunar Tables.*

TABLE I. *The mean Times of New and Full Moon in January, from* A. D. 1700 *to* A. D. 1800, *according to the Old Stile.*

Years.		New Moon.					Full Moon.				
		D.	h.	m.	A	B	D.	h.	m.	A	B
L.	1700	8	0	43	202	1	22	19	5	217	194
	1701	26	22	16	221	336	12	3	54	206	143
	1702	16	7	5	210	286	1	12	43	195	93
	1703	5	15	53	199	236	20	10	15	214	69
L.	1704	23	13	26	218	211	8	19	4	203	18
	1705	12	22	15	207	161	27	16	37	222	354
	1706	2	7	3	196	111	17	1	25	211	304
	1707	21	4	36	215	87	6	10	14	200	254
L.	1708	9	13	24	203	36	24	7	46	218	229
	1709	28	10	57	222	12	15	16	35	207	179
	1710	17	19	46	211	322	3	1	24	196	129
	1711	7	4	34	200	272	21	22	56	215	105
L.	1712	25	2	7	219	247	10	7	45	204	55
	1713	14	10	56	208	197	29	5	18	223	30
	1714	3	19	44	197	147	18	4	6	212	340
	1715	22	17	17	216	122	7	22	55	201	290
L.	1716	11	2	5	205	72	25	20	27	220	265
	1717	0	10	54	194	22	15	5	16	209	215
	1718	19	8	27	213	358	4	14	5	198	165
	1719	8	17	15	202	308	23	11	37	217	140
L.	1720	26	14	48	220	283	11	20	26	205	90
	1721	15	23	37	209	233	1	5	15	194	40
	1722	5	8	25	198	183	20	2	47	213	16
	1723	24	5	58	217	158	9	11	36	202	325
L.	1724	12	14	47	206	108	27	9	9	221	301
	1725	1	23	35	195	58	16	17	57	210	251
	1726	20	21	8	214	34	6	2	46	199	201
	1727	10	5	56	203	344	25	0	18	218	177
L.	1728	28	3	29	222	319	13	9	7	207	126
	1729	17	12	18	211	269	2	17	56	196	76
	1730	6	21	6	200	219	21	15	28	215	52
	1731	25	18	38	219	194	11	0	16	204	1
L.	1732	14	3	28	208	144	28	21	49	223	337

TABLE I. (*Mean Times of New and Full Moon in January, Old Stile*) *continued.*

Years.		New Moon.					Full Moon.				
		D.	h.	m.	A	B	D.	h.	m	A	B
	1733	3	12	16	197	94	18	6	38	212	287
	1734	22	9	49	216	69	7	15	27	201	236
	1735	11	18	37	205	19	26	12	59	220	212
L.	1736	0	3	26	194	329	14	21	48	209	162
	1737	19	0	59	213	305	4	6	37	198	112
	1738	8	9	47	202	254	23	4	9	217	87
	1739	27	7	20	220	230	12	12	58	205	37
L.	1740	15	16	9	209	180	0	21	47	194	347
	1741	5	0	57	198	130	19	19	19	213	323
	1742	23	22	30	217	105	9	4	8	202	273
	1743	13	7	18	206	55	28	1	40	221	248
L.	1744	1	16	7	195	5	16	20	29	210	198
	1745	20	13	40	214	341	5	19	18	199	148
	1746	9	22	28	203	291	24	16	50	218	124
	1747	28	20	1	222	266	14	1	39	207	73
L.	1748	17	4	50	211	216	2	10	28	196	23
	1749	6	13	38	200	166	21	8	0	215	359
	1750	25	11	11	218	141	10	16	49	203	308
	1751	14	19	59	207	91	0	1	37	192	258
L.	1752	3	4	48	196	41	17	23	10	211	234
	1753	22	2	21	215	16	7	7	59	200	184
	1754	11	11	9	204	326	26	5	31	219	159
	1755	0	19	58	193	276	15	14	20	208	109
L.	1756	18	17	31	212	252	3	23	9	197	59
	1757	8	2	19	201	202	22	20	41	216	34
	1758	26	23	52	220	177	12	5	30	205	344
	1759	16	8	41	209	127	1	14	19	194	294
L.	1760	4	17	29	198	77	19	11	51	213	270
	1761	23	15	2	217	52	8	20	40	202	219
	1762	12	23	50	206	2	27	18	12	221	195
	1763	2	8	39	195	312	17	3	1	210	145
L.	1764	20	6	12	214	287	5	11	50	199	95
	1765	9	15	0	203	237	24	9	22	218	70
	1766	28	12	33	221	213	13	18	11	206	20

TABLE I. *(Mean Times of New and Full Moon in January, Old Stile) concluded.*

Years.	New Moon.					Full Moon.				
	D.	h.	m.	A	B	D.	h.	m.	A	B
1767	17	21	22	210	163	3	3	0	195	330
L. 1768	6	6	10	199	113	21	0	32	214	306
1769	25	3	43	218	88	10	9	21	203	255
1770	14	12	31	207	38	29	6	53	222	231
1771	3	21	20	196	348	18	15	42	211	181
L. 1772	21	18	53	215	323	7	0	31	200	131
1773	11	3	41	204	273	25	22	3	219	106
1774	0	12	30	193	223	15	6	52	208	56
1775	19	10	3	212	198	4	15	41	197	6
L. 1776	7	18	51	201	148	22	13	13	216	341
1777	26	16	24	220	124	11	22	2	205	291
1778	16	1	12	209	74	1	6	50	194	241
1779	5	10	1	198	24	20	4	23	213	217
L. 1780	23	7	34	217	359	8	13	12	202	167
1781	12	16	22	206	309	27	10	44	221	142
1782	2	1	11	195	259	16	19	33	210	92
1783	20	22	44	213	234	6	4	22	198	42
L. 1784	9	7	32	202	184	24	1	54	217	17
1785	28	5	5	221	160	13	10	43	206	327
1786	17	13	54	210	110	2	19	32	195	277
1787	6	22	42	199	59	21	17	4	214	252
L. 1788	24	20	15	218	35	10	1	53	203	202
1789	14	5	3	207	345	28	23	25	222	178
1790	3	13	52	196	295	18	8	14	211	128
1791	22	11	25	215	270	7	17	3	200	78
L. 1792	10	20	13	204	220	25	14	35	219	53
1793	0	5	2	193	170	14	23	24	208	3
1794	19	2	35	212	145	4	8	13	197	313
1795	8	11	23	20	95	23	5	45	216	288
L. 1796	26	8	56	219	71	11	14	34	204	238
1797	15	17	44	208	21	0	23	22	193	188
1798	5	2	33	197	331	19	20	55	212	164
1799	24	0	6	216	306	9	5	44	201	114
L. 1800	12	8	54	205	256	27	3	16	220	89

TABLE II. *The Mean Times of New and Full Moon in January, from* A. D. 1752 *to* A. D. 1800, *according to the New Stile.*

Years.	New Moon.					Full Moon.				
	D.	h	m.	A	B	D.	h.	m.	A	B
1753	3	13	37	186	350	18	7	59	200	184
1754	22	11	9	204	326	7	16	47	189	133
1755	11	19	58	193	276	26	14	20	208	109
L. 1756	29	17	31	212	252	14	23	9	197	59
1757	19	2	19	201	201	4	7	57	186	9
1758	8	11	8	190	151	23	5	30	205	344
1759	27	8	41	209	127	12	14	19	194	294
L. 1760	15	17	29	198	77	0	23	7	183	244
1761	5	2	18	187	27	19	20	40	202	219
1762	23	23	50	206	2	9	5	28	191	169
1763	13	8	39	195	312	28	3	1	210	145
L. 1764	1	17	28	184	262	16	11	50	199	95
1765	20	15	0	203	237	5	20	38	188	45
1766	9	23	49	202	187	24	18	11	206	20
1767	28	21	22	210	163	14	3	0	195	330
L. 1768	17	6	10	199	113	2	11	48	184	280
1769	6	14	59	188	63	21	9	21	203	255
1770	25	12	31	207	38	10	18	9	192	205
1771	14	21	20	196	348	0	2	58	181	155
L. 1772	3	6	9	185	298	18	0	31	200	131
1773	22	3	41	204	273	7	9	19	189	80
1774	11	12	30	193	223	26	6	52	208	56
1775	0	21	19	182	173	15	15	41	197	6
L. 1776	18	18	51	201	148	4	0	29	186	316
1777	8	3	40	190	198	22	22	2	205	191
1778	27	1	12	209	74	12	6	50	194	241
1779	16	10	1	98	24	1	15	39	183	191
L. 1780	4	18	50	187	334	19	13	12	202	167
1781	23	16	22	206	309	8	22	0	191	117
1782	13	1	11	195	259	27	19	33	210	92
1783	2	10	0	184	209	17	4	22	198	42
L. 1784	20	7	32	203	184	5	13	10	187	352
1785	9	16	21	192	134	24	10	43	206	327

TABLE II. *concluded. New Stile.*

Years.	New Moon.					Full Moon.				
	D.	h.	m.	A	B	D.	h.	m.	A	B
1786	28	13	54	210	110	13	19	32	195	277
1787	17	22	42	199	59	3	4	20	184	227
L. 1788	6	7	31	188	9	21	1	53	203	202
1789	25	5	3	207	345	10	10	41	192	152
1790	14	13	52	196	295	29	8	14	211	128
1791	3	22	41	185	245	18	17	3	200	78
L. 1792	21	20	13	204	220	7	1	51	189	28
1793	11	5	2	193	170	25	23	24	208	3
1794	0	13	51	182	120	15	8	13	197	313
1795	19	11	23	201	95	4	17	1	186	263
L. 1796	7	20	12	190	45	22	14	34	204	238
1797	26	17	44	208	21	11	23	22	193	188
1798	16	2	33	197	331	1	8	11	182	138
1799	5	11	22	186	281	20	5	44	201	114
C. 1800	24	8	54	205	256	9	14	32	190	63

TABLE III. *Mean Lunations.*

Lun.	D.	h.	m.	A	B	Months.	Days.	
						For	*Subt.*	
1	29	12	44	29	26	January -	- 0	Days to be ſubtracted.
2	59	1	28	58	52	February -	- 31	
3	88	14	12	87	77	March - -	59	
4	118	2	56	116	103	April - -	- 90	
5	147	15	40	146	129	May - -	- 120	
6	177	4	24	175	155	June - -	- 151	
7	206	17	8	204	181	July - -	- 181	
8	236	5	52	233	207	Auguſt -	- 212	
9	265	18	36	262	232	September	- 243	
10	295	7	20	291	258	October -	- 273	
11	324	20	5	320	284	November	- 304	
12	354	8	49	349	310	December	- 334	
13	383	21	33	18	336			

In Leap years, in January and February, add a day to the time found by theſe Tables

TABLE IV. *The firſt Equation.* (A)

A	h.	m.	A	h.	m.	A	h.	m.	A	h.	m.	A	h.	m.
1	4	7	37	1	43	73	0	13	109	0	12	145	1	45
2	4	2	38	1	39	74	0	12	110	0	14	146	1	48
3	3	58	39	1	36	75	0	11	111	0	15	147	1	52
4	3	54	40	1	32	76	0	10	112	0	17	148	1	56
5	3	50	41	1	29	77	0	9	113	0	18	149	1	59
6	3	46	42	1	26	78	0	8	114	0	20	150	2	3
7	3	42	43	1	23	79	0	7	115	0	22	151	2	7
8	3	37	44	1	19	80	0	6	116	0	23	152	2	11
9	3	33	45	1	16	81	0	5	117	0	25	153	2	15
10	3	29	46	1	13	82	0	4	118	0	27	154	2	19
11	3	24	47	1	10	83	0	3	119	0	29	155	2	23
12	3	20	48	1	7	84	0	2	120	0	31	156	2	27
13	3	16	49	1	4	85	0	2	121	0	34	157	2	31
14	3	12	50	1	1	86	0	1	122	0	36	158	2	35
15	3	7	51	0	58	87	0	1	123	0	38	159	2	39
16	3	3	52	0	56	88	0	1	124	0	40	160	2	43
17	2	59	53	0	53	89	0	0	125	0	43	161	2	48
18	2	55	54	0	51	90	0	0	126	0	45	162	2	52
19	2	51	55	0	48	91	0	0	127	0	48	163	2	56
20	2	47	56	0	45	92	0	0	128	0	51	164	3	0
21	2	43	57	0	43	93	0	0	129	0	53	165	3	5
22	2	39	58	0	40	94	0	0	130	0	55	166	3	9
23	2	35	59	0	38	95	0	1	131	0	59	167	3	13
24	2	31	60	0	36	96	0	1	132	1	2	168	3	18
25	2	27	61	0	34	97	0	1	133	1	5	169	3	22
26	2	23	62	0	32	98	0	2	134	1	8	170	3	27
27	2	19	63	0	30	99	0	2	135	1	11	171	3	31
28	2	15	64	0	28	100	0	3	136	1	14	172	3	35
29	2	11	65	0	26	101	0	4	137	1	17	173	3	40
30	2	8	66	0	24	102	0	5	138	1	20	174	3	44
31	2	4	67	0	22	103	0	6	139	1	24	175	3	49
32	2	0	68	0	20	104	0	7	140	1	27	176	3	53
33	1	57	69	0	19	105	0	8	141	1	30	177	3	58
34	1	54	70	0	17	106	0	9	142	1	34	178	4	2
35	1	50	71	0	15	107	0	10	143	1	37	179	4	7
36	1	46	72	0	14	108	0	11	144	1	41	180	4	11

TABLE IV. (*Equation* A) *concluded.*

A	h.	m.	A	h.	m.	A	h.	m.	A	h.	m.	A	h.	m.
181	4	15	217	6	45	253	8	12	289	8	7	325	6	32
182	4	20	218	6	48	254	8	13	290	8	5	326	6	29
183	4	24	219	6	52	255	8	14	291	8	3	327	6	25
184	4	29	220	6	55	256	8	15	292	8	2	328	6	22
185	4	33	221	6	58	257	8	16	293	8	0	329	6	18
186	4	38	222	7	2	258	8	17	294	7	58	330	6	14
187	4	42	223	7	5	259	8	18	295	7	56	331	6	10
188	4	47	224	7	8	260	8	19	296	7	54	332	6	7
189	4	51	225	7	11	261	8	20	297	7	52	333	6	3
190	4	55	226	7	14	262	8	20	298	7	50	334	5	59
191	5	0	227	7	17	263	8	21	299	7	48	335	5	55
192	5	4	228	7	20	264	8	21	300	7	46	336	5	51
193	5	9	229	7	23	265	8	21	301	7	44	337	5	47
194	5	13	230	7	26	266	8	22	302	7	41	338	5	43
195	5	17	231	7	29	267	8	22	303	7	39	339	5	39
196	5	22	232	7	31	268	8	22	304	7	37	340	5	35
197	5	26	233	7	34	269	8	22	305	7	34	341	5	31
198	5	30	234	7	37	270	8	22	306	7	31	342	5	27
199	5	34	235	7	39	271	8	22	307	7	29	343	5	22
200	5	39	236	7	41	272	8	21	308	7	26	344	5	19
201	5	43	237	7	44	273	8	21	309	7	23	345	5	15
202	5	47	238	7	46	274	8	20	310	7	21	346	5	10
203	5	51	239	7	48	275	8	20	311	7	18	347	5	6
204	5	55	240	7	50	276	8	19	312	7	15	348	5	2
205	5	59	241	7	53	277	8	19	313	7	12	349	4	58
206	6	3	242	7	55	278	8	18	314	7	9	350	4	54
207	6	7	243	7	57	279	8	17	315	7	6	351	4	49
208	6	11	244	7	59	280	8	16	316	7	3	352	4	45
209	6	15	245	8	0	281	8	15	317	7	0	353	4	41
210	6	19	246	8	2	282	8	14	318	6	56	354	4	37
211	6	23	247	8	4	283	8	13	319	6	53	355	4	34
212	6	26	248	8	6	284	8	12	320	6	50	356	4	28
213	6	30	249	8	7	285	8	11	321	6	46	357	4	24
214	6	34	250	8	9	286	8	10	322	6	43	358	4	20
215	6	37	251	8	10	287	8	9	323	6	39	359	4	15
216	6	41	252	8	11	288	8	8	324	6	36	360	4	11

TABLE V. *The ſecond Equation.* (B)

B	h.	m.	B	h.	m.	B	h.	m.	B	h.	m.	B	h.	m.
1	9	59	37	16	2	73	19	21	109	18	51	145	15	4
2	10	10	38	16	11	74	19	23	110	18	47	146	14	56
3	10	21	39	16	19	75	19	25	111	18	43	147	14	48
4	10	32	40	16	27	76	19	27	112	18	38	148	14	39
5	10	43	41	16	35	77	19	29	113	18	34	149	14	31
6	10	54	42	16	43	78	19	30	114	18	29	150	14	23
7	11	5	43	16	50	79	19	32	115	18	24	151	14	14
8	11	16	44	16	58	80	19	33	116	18	19	152	14	5
9	11	27	45	17	5	81	19	34	117	18	14	153	13	57
10	11	38	46	17	12	82	19	35	118	18	8	154	13	48
11	11	49	47	17	19	83	19	35	119	18	3	155	13	39
12	11	59	48	17	26	84	19	36	120	17	57	156	13	31
13	12	10	49	17	33	85	19	36	121	17	51	157	13	22
14	12	20	50	17	39	86	19	36	122	17	45	158	13	13
15	12	31	51	17	46	87	19	36	123	17	40	159	13	4
16	12	42	52	17	52	88	19	36	124	17	34	160	12	55
17	12	52	53	17	58	89	19	35	125	17	28	161	12	46
18	13	2	54	18	4	90	19	35	126	17	22	162	12	37
19	13	13	55	18	9	91	19	34	127	17	15	163	12	28
20	13	23	56	18	15	92	19	33	128	17	9	164	12	18
21	13	33	57	18	20	93	19	32	129	17	2	165	12	9
22	13	43	58	18	25	94	19	31	130	16	56	166	12	0
23	13	53	59	18	30	95	19	30	131	16	49	167	11	51
24	14	3	60	18	35	96	19	28	132	16	42	168	11	41
25	14	13	61	18	40	97	19	26	133	16	35	169	11	32
26	14	23	62	18	44	98	19	24	134	16	28	170	11	23
27	14	32	63	18	48	99	19	22	135	16	21	171	11	14
28	14	42	64	18	52	100	19	20	136	16	14	172	11	4
29	14	52	65	18	56	101	19	18	137	16	6	173	10	55
30	15	1	66	19	0	102	19	15	138	15	59	174	10	45
31	15	10	67	19	4	103	19	12	139	15	51	175	10	36
32	15	19	68	19	7	104	19	9	140	15	44	176	10	26
33	15	28	69	19	10	105	19	6	141	15	36	177	10	17
34	15	37	70	19	13	106	19	2	142	15	28	178	10	7
35	15	45	71	19	16	107	18	59	143	15	20	179	9	58
36	15	54	72	19	19	108	18	55	144	15	12	180	9	48

TABLE V. (*Equation* B) *concluded.*

B	h.	m.	B	h.	m.	B	h.	m.	B	h.	m.	B	h.	m.
181	9	38	217	4	16	253	0	38	289	0	20	325	3	51
182	9	29	218	4	8	254	0	35	290	0	23	326	3	59
183	9	19	219	4	0	255	0	31	291	0	26	327	4	8
184	9	10	220	3	52	256	0	27	292	0	29	328	4	17
185	9	0	221	3	45	257	0	24	293	0	32	329	4	26
186	8	51	222	3	37	258	0	21	294	0	36	330	4	35
187	8	41	223	3	30	259	0	19	295	0	40	331	4	44
188	8	32	224	3	22	260	0	16	296	0	44	332	4	54
189	8	22	225	3	15	261	0	14	297	0	48	333	5	2
190	8	13	226	3	8	262	0	12	298	0	52	334	5	13
191	8	4	227	3	1	263	0	10	299	0	56	335	5	23
192	7	55	228	2	54	264	0	8	300	1	1	336	5	33
193	7	45	229	2	47	265	0	6	301	1	6	337	5	43
194	7	36	230	2	40	266	0	5	302	1	11	338	5	53
195	7	27	231	2	34	267	0	4	303	1	16	339	6	3
196	7	18	232	2	27	268	0	3	304	1	22	340	6	13
197	7	8	233	2	21	269	0	2	305	1	27	341	6	23
198	6	59	234	2	14	270	0	1	306	1	32	342	6	34
199	6	50	235	2	8	271	0	1	307	1	38	343	6	44
200	6	41	236	2	2	272	0	1	308	1	44	344	6	54
201	6	32	237	1	56	273	0	0	309	1	50	345	7	5
202	6	23	238	1	51	274	0	0	310	1	57	346	7	16
203	6	14	239	1	45	275	0	0	311	2	3	347	7	26
204	6	5	240	1	39	276	0	1	312	2	10	348	7	37
205	5	57	241	1	33	277	0	1	313	2	17	349	7	48
206	5	48	242	1	28	278	0	2	314	2	24	350	7	58
207	5	39	243	1	22	279	0	3	315	2	31	351	8	2
208	5	31	244	1	17	280	0	4	316	2	38	352	8	20
209	5	22	245	1	12	281	0	6	317	2	46	353	8	31
210	5	13	246	1	7	282	0	7	318	2	53	354	8	42
211	5	5	247	1	2	283	0	9	319	3	1	355	8	53
212	4	57	248	0	58	284	0	11	320	3	9	356	9	4
213	4	49	249	0	53	285	0	13	321	3	17	357	9	15
214	4	40	250	0	49	286	0	14	322	3	25	358	9	26
215	4	32	251	0	46	287	0	16	323	3	34	359	9	37
216	4	24	252	0	42	288	0	17	324	3	42	360	9	48

TABLE VI. *Supplemental to* TABLE I. *for finding the mean Time of New or Full Moon in January for 6000 Years before or after any given Year in the 18th Century, according to the Julian or Old Stile.*

Years.	D.	h.	m.	A	B	Years.	D.	h.	m	A	B
100	4	8	11	3	255	3100	16	10	41	347	253
200	8	16	22	7	151	3200	20	18	52	351	148
300	13	0	33	10	46	3300	25	3	3	354	44
400	17	8	43	13	301	3400	29	11	14	357	299
500	21	16	54	17	197	3500	4	6	40	332	169
600	26	1	5	20	92	3600	8	14	51	335	64
700	0	20	32	354	322	3700	12	23	2	338	319
800	5	4	43	358	217	3800	17	7	13	342	215
900	9	12	54	1	112	3900	21	15	24	345	110
1000	13	21	5	4	8	4000	25	23	35	349	6
1100	18	5	16	8	263	4100	0	19	1	323	235
1200	22	13	26	11	159	4200	5	3	12	326	130
1300	26	21	37	14	54	4300	9	11	23	329	26
1400	1	17	4	349	284	4400	13	19	34	333	281
1500	6	1	15	352	179	4500	18	3	45	336	177
1600	10	9	26	355	74	4600	22	11	56	340	72
1700	14	17	37	359	330	4700	26	20	7	343	327
1800	19	1	48	2	225	4800	1	15	33	317	197
1900	23	9	58	5	120	4900	5	23	44	321	92
2000	27	18	9	9	16	5000	10	7	55	324	348
2100	2	13	36	343	245	5100	14	16	6	327	243
2200	6	21	47	346	141	5200	19	0	17	331	138
2300	11	5	58	350	36	5300	23	8	28	334	34
2400	15	14	9	353	291	5400	27	16	39	337	289
2500	19	22	20	356	187	5500	2	12	5	312	159
2600	24	6	31	0	82	5600	6	20	16	315	54
2700	28	14	41	3	337	5700	11	4	27	318	309
2800	3	10	8	337	207	5800	15	12	38	322	205
2900	7	18	19	341	102	5900	19	20	49	325	100
3000	12	2	30	344	358	6000	24	5	0	328	355

To calculate the true Time of New or Full Moon.

For any propoſed year, within the limits of Table I. for Old Stile, or Table II. for New Stile, write out the mean time of New or Full Moon in January, with the numbers or arguments under A and B. With theſe arguments find the equations in Tab. IV. and V. which being added to the mean time of New or Full Moon in January, will give the true time thereof in that month.

For the time of New or Full Moon in any month after January, add as many lunations from Table III. to the mean time in January, as the given month is after January, and alſo the numbers A and B for theſe lunations, to the numbers A and B belonging to the mean time in January. Then, with the reſpective ſums of theſe numbers, if under 360, find the correſponding equations in Tab. IV. and V. which added to the mean time will give the true time of the required New or Full Moon, in days, hours, and minutes, from the beginning of January. When either of the ſums A or B exceeds 360, ſubtract 360 from it; and with the remainder enter the correſponding Table, and take out the equation.

Then, from the number of days made by the lunations added to the New or Full Moon day in January, ſubtract the number of days againſt the given month in Table III. and the remainder will be the day of the required New or Full Moon in that month. If this number of days be equal to the number you ſubtract them from, the required New or Full Moon falls not in the given month, but on the laſt day of the month preceding it: and, when that is the caſe, you muſt add a lunation to it from Table III. with the equations A and

B for that lunation; and then you will have the true time of New or Full Moon in the given month.

If the number of the given month after January be equal to or exceed the number of days accounted from the beginning of January on which the New or Full Moon therein falls, you muſt take out one lunation more from Table III. than what anſwers to the number of the given month after January; otherwiſe you would have the New or Full Moon not in the month you want, but in the month next before it. And this will always be the caſe in ſome month or other of the year wherein the New or Full Moon in January falls before the 11th day thereof.

In leap years, in the months of January and February, the Tables give the New or Full Moon a day ſooner than it really falls: and therefore, in theſe years and months, a day muſt be added to the time found by theſe Tables. They always begin the day at noon, and reckon the hours onward from that time to the noon of the following day.

Example I. *For the true Time of New Moon in June* A. D. 1772, *New Stile.*

	D.	h.	m.	A	B
To New Moon in Jan. 1772, Tab. II.	3	6	9	185	298
Add 6 lunations from Tab. III. -	177	4	24	175	155
The ſums are - - - -	180	10	33	360	453
Firſt equation (A) for 360, Tab. IV.		4	11		360
Second equation (B) for 93, Tab. V.		19	32		93
The whole makes - - -	181	10	10		
Table III. againſt June, ſubtract	151				
Remains the true time, *viz.* June	30	10	16		

So the true time of the required New Moon is the 30th of June, at 16 minutes paſt X in the evening.

Example II. *For the true Time of Full Moon in June* A. D. 1772, *New Stile.*

	D.	h.	m	A	B
To Full Moon in Jan. 1772, Tab. II.	18	0	31	200	131
Add 5 lunations from Tab. III. -	147	15	40	146	129
The ſums are - - - -	165	16	11	346	260
Firſt equation (A) for 346, Tab. IV.		5	10		
Second equation (B) for 260, Tab. V.		0	16		
The whole makes - - -	165	21	37		
Table III. againſt June, ſubtract	151				
Remains the true time, *viz.* June	14	21	37		

Namely, the 15th of June, at 37 minutes paſt IX in the morning.

To calculate the true Time of New or Full Moon in any given Year and Month after the 18th century, which begins with A. D. 1700, *and ends with* A. D. 1800.

Here we muſt go by the Old Stile, and then reduce the time to the New, by the Table farther on, ſhewing the number of days whereby the ſtiles do differ.——In Tab. I. find a year in the 18th century of the ſame number with that in the century propoſed: then from that year, in the 18th century, write out the mean time of New or Full Moon in January, with the numbers A and B belonging thereto. This done, find a year in Table VI. which, when added to the ſaid year in the 18th century, ſhall make up the number of the given year. Take out the time and numbers A and B for that year in Table VI. and add them to thoſe in January in Table I. and the ſums ſhall be the mean time of New or Full Moon in January for the given year, and the arguments for finding the equations in that month. Then, for any other month in the given year, work as already taught.

When the ſum of days for January exceeds 31 (as in the following example) ſubtract a lunation (Tab. III.) from the time, and alſo the numbers A and B for that lunation from the former numbers, and ſet down the remainders for the mean time in January, and equation-arguments for that month. If either of the numbers A or B to be ſubtracted be greater than the number you would ſubtract from, add 360 to the leſſer number, and then make the ſubtraction.

Example III. *For the true Time of New Moon in May* A. D. 1909, *Old Stile.*

To A. D. 1709 add 200, and the ſum will be 1909.

	D.	h.	m.	A	B
To New Moon in Jan. 1709, Tab. I.	28	10	57	222	12
Add, for 200 years, from Tab. VI.	8	16	22	7	151
The ſums are - - - -	37	3	19	229	163
Subtract 1 lunation, Tab. III. -	29	12	44	29	26
Rem. mean New Moon in Jan. 1909	7	14	35	200	137
Add 4 lunations from Tab. III.	118	2	5	116	103
The ſums are - - - -	125	17	31	316	240
Firſt equation (A) for 316, Tab. IV.		7	3		
Second equation (B) for 240, Tab. V.		1	39		
The whole makes - - -	126	2	13		
Table III. againſt May, ſubtract	120				
Remains the true time, *viz.* May	6	2	13		

So the true time of the required New Moon is the 6th of May, at 13 minutes paſt II in the afternoon.

There was no difference between the Old Stile and the New in the year of Chriſt 200.—In any century after that year, to find the difference between the Old and New Stile, divide the number of the given century by 4, and (without regarding the remainder when there is any) add 3 to the quotient; then ſubtract the ſum from the number of the century, and the remainder ſhall be the number of days which muſt be added to the Old Stile time, to reduce it to the New. Thus, in the year 1909, the difference will be found to be 12 days: and therefore, in the preceding example, the day of New Moon in May will be the 18th, according to the New Stile. —At the end of theſe precepts and examples, I ſhall ſubjoin a Table of theſe differences, which I have copied from a certain Author, who has copied much more from my book of Aſtronomy into one of his, without doing the common juſtice of acknowledging, in theſe caſes, from whom he copied.

To calculate the true Time of New or Full Moon in any given Year and Month between the Chriſtian æra and the 18th Century, Old Stile.

Find a year in the 18th century of the ſame number with that in the century propoſed, and take out the mean time of New or Full Moon in January, from Table I. for the ſaid year in the 18th century, with its numbers A and B. Then, from Table VI. take out the time and numbers A and B for as many years as, when ſubtracted from the above-mentioned year in the 18th century, ſhall leave the given year remaining. Subtract the times and numbers taken from Table IV. from thoſe taken from Table I. and ſet down the remainders for the mean time of New or Full Moon in January in the given year, and the arguments A and B for finding the equations in January: and then, for any other month in that year, work as above taught.

When ſubtraction of days cannot be made (as in the following example) add a lunation, and its numbers A and B, from Table III. to the firſt time and numbers taken out: then ſubtract, and ſet down the remainders for the mean time of New Moon in January, and the arguments for finding the equations belonging to it.

Example IV. *For the true Time of Full Moon in April, Old Stile, in the Year of Chriſt* 796.

From A. D. 1796 ſubtract 1000, and 796 will remain.

	D.	h.	m.	A	B
To Full Moon in Jan. A. D. 1796	11	14	34	204	238
Add 1 lunation from Table III.	29	12	44	29	26
The ſums are - - - -	41	3	18	233	264
From which ſubt. for 1000 years, Table VI. - - -	13	21	5	4	8
Rem. for Full Moon, Jan. A. D. 796	27	6	13	229	256
Add 3 lunations, for April, Tab. III.	88	14	12	87	77
The ſums are - - - -	115	20	25	316	333
Firſt equation (A) for 316, Tab. IV.		7	3		
Second equation (B) for 333, Tab. V.		5	2		
The whole makes - - -	116	8	30		
Table III. for April, ſubtract -	90				
Remains the true time, *viz.* April	26	8	30		

So that, in April A. D. 796, the true time of Full Moon was the 28th day, at 30 min. paſt VIII in the evening.

To calculate the true Time of New or Full Moon in any given Year and Month before the Chriſtian æra, according to the Old Stile.

Find a year in the 18th century, which being added to the given number of years before Chriſt diminiſhed by one, ſhall make any number of compleat centuries.

Find this number of centuries in Table VI. and ſubtract the time and numbers A and B belonging to it from thoſe in the year of the 18th century; and the remainder will be the mean time of New or Full Moon for January in the given year, and the numbers A and B belonging thereto. Then, for any other month in that year, work as above.

When the number of days taken from Tab. VI. exceeds the number taken from Tab. I. add a lunation. And when ſubtraction cannot be made in the numbers A or B (as is the caſe in the 3d and 4th lines (A) in the following example) add 360 to the leſſer number, and then make ſubtraction.

EXAMPLE V. *For the true Time of New Moon in May, Old Stile, the Year before Chriſt* 585.

The years 584 added to 1716, make 2300, or 23 centuries.

	D.	h.	m.	A	B
To New Moon in Jan 1716, TAB. I.	11	2	5	205	72
Add 1 lunation from TABLE III. -	29	12	44	29	26
The ſums are - - - - -	40	14	49	234	98
Subtract from TAB. VI. for 2300 years	11	5	58	350	36
New Moon in Jan. bef. Chriſt 585	29	8	51	244	62
Add 4 lunations, for April, TAB. III.	118	2	56	116	103
The ſums are, for April - -	147	11	47	360	105
Firſt equation (A) for 360, TAB. IV.		4	11		
Second equation (B) for 165, TAB. V.		12	9		
The whole makes - - -	148	4	7		
TABLE III. for May, ſubtract -	120				
Remains the true time, *viz.* May	28	4	7		

So that the true time was the 28th of May, at 7 minutes paſt IV in the afternoon.—At that time, a total eclipſe of the Sun put an end to the long war between the Medes and Lydians, by frightening both the armies with a ſudden darkneſs, which overſpread the field of battle, juſt as they were ready to begin the deciſive engagement.

Some chronologiſts believe, that the world was created at the time of the autumnal equinox, in October, the year 4007 before the year of Chriſt's birth, and that the Moon was full upon the third day of the creation week, which muſt have been Wedneſday.—But, by a calculation from theſe Tables, I find that the ſaid Full-Moon was on Tueſday, the 23d of October, at 45 minutes paſt VI in the morning; and Dr. Halley's Tables make it but one minute ſooner. In that year, the autumnal equinox was on Wedneſday, October the 24.

Theſe Tables are adapted to the meridian of London; but they will anſwer for any other, by adding 4 minutes to the time found by them for every degree eaſtward from the meridian of London, or ſubtracting 4 minutes for every degree weſtward. They are as near the truth as can be, by having only two equations; and I have never found them to differ half an hour from Meyer's, which have thirteen.

And thus, by a very ſhort and eaſy method, the time of any New or Full Moon, within the limits of 6000 years either before or after the Chriſtian æra, may be found, ſufficiently near the truth for any common purpoſe.—I have tried various methods for making ſuch tables as ſhould render the calculations of New and Full Moons from them ſtill ſhorter, but have hitherto found it impoſſible, unleſs I ſhould have contented myſelf with ſuch as would ſometimes vary a whole hour from the truth.

A TABLE *ſhewing the Number of Days between the Old and New Stile in different Periods of Time.*

Years before Chriſt. New Stile.	Days Diff. —	Years before Chriſt. New Stile.	Days Diff. — +	Years after Chriſt. New Stile.	Day Diff. +
L. 6000	47	L. 2000	17	L. 2000	13
5900	46	1900	16	2100	14
5800	45	1800	15	2200	15
5700	44	1700	14	2300	16
L. 5600	44	L. 1600	14	L. 2400	16
5500	43	1500	13	2500	17
5400	42	1400	12	2600	18
5300	41	1300	11	2700	19
L. 5200	41	L. 1200	11	L. 2800	19
5100	40	1100	10	2900	20
5000	39	1000	9	3000	21
4900	38	900	8	3100	22
L. 4800	38	L. 800	8	L. 3200	22
4700	37	700	7	3300	23
4600	36	600	6	3400	24
4500	35	500	5	3500	25
L. 4400	35	L. 400	5	L. 3600	25
4300	34	300	4	3700	26
4200	33	200	3	3800	27
4100	32	*Bef.* 100	2	3900	28
L. 4000	32	L. 0	2	L. 4000	28
3900	31	*Aft.* 100	1	4100	29
3800	30	200	— + 0	4200	30
3700	29	300	1	4300	31
L. 3600	29	L. 400	1	L. 4400	31
3500	28	500	2	4500	32
3400	27	600	3	4600	33
3300	26	700	4	4700	34
L. 3200	26	L. 800	4	L. 4800	34
3100	25	900	5	4900	35
3000	24	1000	6	5000	36
2900	23	1100	7	5100	37
L. 2800	23	L. 1200	7	L. 5200	37
2700	22	1300	8	5300	38
2600	21	1400	9	5400	39
2500	20	1500	10	5500	40
L. 2400	20	L. 1600	10	L. 5600	40
2300	19	1700	11	5700	41
2200	18	1800	12	5800	42
2100	17	1900	13	5900	43

The days in the columns marked — are to be ſubtracted from the Old Stile time to reduce it to the New; and thoſe in the columns marked + are to be added to the Old Stile, in order to reduce it to the New. All the years in this Table are Leap years in the Old Stile, but only thoſe which are marked L are Leap years in the New. The difference between the two Stiles was nothing in the 200dth year after the year of Chriſt's birth: from that time, backward or forward, it varies 3 days in every 400 years.

The Weight of Gold compared with the Weights of other Materials, of equal Bulk with the Gold.

An hundred pound weight of pure fine Gold is equal in bulk to	An hundred pound weight of coinage Gold is equal in bulk to
Pound weight.	Pound weight.
90.3 of Guinea Gold.	110.8 of pure fine Gold.
69.1 of Quickſilver.	76.6 of Quickſilver.
58.1 of Lead.	64.4 of Lead.
56.5 of pure Silver.	62.6 of pure Silver.
54.1 of coinage Silver.	59.4 of coinage Silver.
49.4 of Biſmuth.	54.7 of Biſmuth.
45.8 of Copper.	50.8 of Copper.
44.1 of hammered Braſs.	48.3 of hammered Braſs.
40.3 of caſt Braſs.	44.6 of caſt Braſs.
40.2 of forged Iron.	44.5 of forged Iron.
39.8 of Spring Steel.	44.0 of Spring Steel.
37.2 of Block Tin.	41.2 of Block Tin.
35.7 of Silver Ore.	39.5 of Silver Ore.
30.5 of Gold Litharge.	33.8 of Gold Litharge.
25.5 of Lapis Calaminaris.	28.2 of Lapis Calaminaris.
24.2 of Loadſtone.	26.8 of Loadſtone.
21.3 of Copper Ore.	23.0 of Copper Ore.
20.8 of Sapphires.	22.5 of Sapphires.
17.9 of Diamonds.	19.9 of Diamonds.
16.2 of Cornelian Stone.	18.4 of Cornelian Stone.
16.0 of Cryſtal Glaſs.	17.7 of Cryſtal Glaſs.
15.5 of Lapis Lazuli.	17.2 of Lapis Lazuli.
15.0 of Plate Glaſs.	16.6 of Plate Glaſs.
14.5 of red Coral.	16.1 of red Coral.
13.8 of Iceland Cryſtal.	15.3 of Iceland Cryſtal.
13.8 of white Marble.	15.3 of white Marble.
13.4 of Rock Cryſtal.	15.0 of Rock Cryſtal.
13.1 of homogeneal Pyrites.	14.6 of homogeneal Pyrites.
11.9 of Sulphur.	13.2 of Sulphur.
9.5 of black Lead.	10.5 of black Lead.
5.4 of Frankincenſe.	6.0 of Frankincenſe.
5.1 of common Water.	5.6 of common Water.
5.0 of Camphire.	5.5 of Camphire.
4.7 of Proof Spirits.	5.2 of Proof Spirits.
4.4 of pure Spirits.	4.9 of pure Spirits.
3.9 of Oil of Turpentine.	4.3 of Oil of Turpentine.
3.7 of Æther.	4.1 of Æther.
1.2 of Cork.	1.3 of Cork.

A TABLE ſhewing the Standard Weight, Value, and comparative View of Engliſh Silver Money from King William the First, A. D. 1066, to A. D. 1765, according to the Mint Indentures.

Dates of the ſeveral Mint Indentures.	Standard of the Silver at each Period.				Number of Shillings and Pence in the Troy Pound of Standard Silver coined at each Period.		Weight of 20 Shillings in reckoning of Standard Silver at each of theſe Periods.			Weight of fine Silver contained in 20 Shillings in reckoning at each particular Period.		
	Fine Silver.		Copper Alloy.									
A. D.	oz.	pw.	oz.	pw.	ſh.	d.	oz.	pw.	gr.	oz.	pw.	gr.
1066	11	2	0	18	21	4	11	5	0	10	8	3
1087	11	2	0	18	20	0	12	0	0	11	2	0
1300	11	2	0	18	20	3	11	17	1	10	19	6
1347	11	2	0	18	22	6	10	13	8	9	17	8
1354 1395 1402	11	2	0	18	25	0	8	12	0	8	17	14
1412	11	2	0	18	32	0	7	10	0	6	18	18
1422	11	2	0	18	30	0	8	0	0	7	8	0
1422	11	2	0	18	37	6	6	8	0	5	18	10
1426 1446	11	2	0	18	30	0	8	0	0	7	8	0
1461 1464 1482 1483 1494	11	2	0	18	37	6	6	8	0	5	18	10
1505	11	2	0	18	40	0	6	0	0	5	11	0
1509 1532	11	2	0	18	45	0	5	6	16	4	18	6
1543	10	0	2	0	48	0	5	0	0	4	3	8
1545	6	0	6	0	48	0	5	0	0	2	10	0
1546 1547 1548	4	0	8	0	48	0	5	0	0	1	13	8
1549	6	0	6	0	72	0	3	6	16	1	13	8
1551	3	0	9	0	72	0	3	6	16	0	16	16
1553	11	1	0	19	60	0	4	0	0	3	13	16
1560 1583	11	2	0	18	60	0	4	0	0	3	14	0
1601 1605 162 1661 1671 1685 1720 1761	11	2	0	18	62	0	3	17	10	3	11	15

The TABLE *concluded.*

Dates of the ſeveral Mint Indentures.	Value of the ſame 20 Sh. in reckoning of our preſent Money.			Proportion of Money at each Period to that of our preſent Money.	Value of the Ounce of the then Standard Silver to that of our preſent Money.		Value of the Ounce of fine Silver at each Period.		Kings and Queens in theſe Periods.
A. D.	l.	s	d.	——	ſh.	d.	ſh.	d.	——
1066	2	18	1½	2.9062	5	2	1	11⅛	Will. Conq.
1087	3	2	0	3.1000	5	2	1	9⅝	Will. Rufus.
1300	3	1	2¾	3.0614	5	2	1	9⅞	Edward I.
1347	2	15	1⅜	2.7557	5	2	2	0⅜	Edward III.
1354 1395 1402	2	9	7¼	2.4802	5	2	2	3	Richard II. Henry IV.
1412	1	18	9	1.9375	5	2	2	10⅝	
1422	2	1	4	2.0666	5	2	2	8½	Henry VI.
1422	1	13	0¾	1.6531	5	2	3	4½	
1426 1446	2	1	4	2.0666	5	2	2	8½	Edward IV.
1461 1464 1482 1483 1494	1	13	0¾	1.6531	5	2	3	4½	Edward V. Henry VII.
1505	1	11	0	1.5500	5	2	3	7¼	
1509 1532	1	7	6⅝	1.3776	5	2	4	0⅝	Henry VIII.
1543	1	3	3¼	1.1635	4	7⅞	4	9⅝	
1545	0	13	11⅝	0.6984	2	9½	8	0	
1546 1547 1548	0	9	3¾	0.4656	1	10⅜	12	0	Edward VI.
1549	0	9	3¾	0.4656	2	9½	12	0	
1551	0	4	7⅞	0.2328	1	4¾	24	0	
1553	1	0	6⅞	1.0286	5	1¾	5	5⅛	Mary I.
1560 1583	1	0	8	1.0333	5	2	5	4⅞	Elizabeth.
1601 1605 1627 1661 1671 1685 1720 1764	1	0	0	1.0000	5	2	5	7	James I. Charles I. Charles II. James II. George I. George III.

Prices of Goods at the above-mentioned Times.

From *A. D.* 1000 to *A. D.* 1066. A horſe 1 l. 17 s. 6 d. A cow 6 s. An ox 7 s. 6 d. A ſwine 2 s. A ſheep 1 s. 3 d. Wheat *per* quarter 1 s. 6 d.

From *A. D.* 1066 to *A. D.* 1199. A horſe 12 s. 5 d. An ox 4 s. 8 d. A ſow 3 s. A colt 2 s. 4½ d. A calf 2 s. 4¼ d. A ſheep 1 s. 8 d. Wheat *per* quarter 3 s. 1 d.

From *A. D.* 1199 to *A. D.* 1307. A horſe 1 l. 11 s. An ox 4 s. 8 d. A ſow 3 s. A cow 17 s. 0½ d. A lamb 4 s. A heifer 2 s. 1½ d. A gooſe 1 s. 0½ d. A cock 4½ d. A hen 3 d. Wheat *per* quarter 1 l. 3 s. 2½ d.

From *A. D.* 1307 to 1418. A horſe 18 s. 4 d. An ox 2 l. 6 s. 1 d. A cow 7 s. 2 d. A calf 4 s. 2 d. A ſheep 2 s. 7 d. A gooſe 9 d. A cock 3¾ d. A hen 2¾ d. Wheat *per* quarter 15 s. Ale *per* gallon 7¼ d. Day labourer's wages 4½ d.

From *A. D.* 1418 to *A. D.* 1524. A horſe 2l. 4s. An ox 1 l. 15s. 8½ d. A cow 15s. 6d. A colt 7 s. 8d. A ſheep 5s. A hog 5s. A calf 4s. 1 d. A cock 3d. A hen 2d. Wheat *per* quarter 11 s. 3d. Ale *per* gallon 2¾ d. Day labourer's wages 3¾ d.

From *A. D.* 1524 to *A. D.* 1604. An ox 1 l. 16s. 7d. A ſheep 4s. 3¾ d. A calf 5 s. 6d. A lamb 4s. 4¼ d. A gooſe 1 s. A capon 1 s. Beef *per* ſtone 11 d. Coals *per* chaldron 7s. 9¼ d. Wheat *per* quarter 15 s.

From *A. D.* 1624 to *A. D.* 1646. A pheaſant 5 s. 6 d. A turkey 3 s. 9d. A gooſe 2 s. A partridge 1 s. A pullet 1 s. 6d. A pigeon 6d.

From *A. D.* 1730 to *A. D.* 1760. A horſe 10 l. An ox 8 l. A cow 7 l. 7 s. A hog 1 l. 15 s. A ſheep 1 l., 6 s. A turkey 4 s. A cock 1 s. 3d. Seamens wages *per* day 9d. Common labourers 1 s. 8 d.

In the preceding Table, by comparing the number of ſhillings in the pound weight of ſilver in the former times with the number in the pound weight at preſent, it will be found that the above-mentioned articles were not ſo cheap as is now generally believed.

Concerning Gold.

The ſtandard for coinage gold is 11 ounces of pure gold and 1 ounce of copper.

At the Tower of London, 44 guineas and an half are coined out of a Troy pound, or 5760 grains, of gold. Hence, the ſtandard weight of a guinea is $129\frac{4382}{100000}$ grains.

The weight of a cubic foot of ſuch gold is $7524921\frac{7}{10}$ grains (or 1306 Troy pounds 4 ounces 18 penny-weight $9\frac{7}{10}$ grains. Out of this quantity, $58135\frac{1}{4}$ guineas may be coined, which is equal in value to 61042 pounds 0 ſhillings 3 pence ſterling.

A lump of this gold, equal in bulk to $2129\frac{7}{10}$ cubic feet, coined into guineas, would pay the national debt.

The Number of different Ways in which all the Letters of the Alphabet might be combined, or put together, from 1 Letter to 25. Or, the Number of Changes which might be rung on any Number of Bells not exceeding the Number of Letters in the Alphabet.

Thus 2 letters may be put 2 different ways together; 3 letters, 6 different ways; 4 letters, 24 ways; 5 letters, 120 ways; 6 letters, 750 ways; and ſo on, as in the following Table:

1	A	1
2	B	2
3	C	6
4	D	24
5	E	120
6	F	720
7	G	5040
8	H	40320
9	I	362880
10	K	3628800
11	L	39916800
12	M	479001600
13	N	6227020800
14	O	87178291200
15	P	1307674368000
16	Q	20922789888000
17	R	355687428096000
18	S	6402373705728000
19	T	121645100408832000
20	U	2432902008176640000
21	V	51090942171709440000
22	W	1124000727777607680000
23	X	25852016738884976640000
24	Y	620448401733239439360000
25	Z	15511210043330985984000000

Now, ſuppoſing all the 25 letters could be put down in 30 ſeconds of time, or each combination of them made in that time (which might be done) it would require 57461442099517020244 Julian years to make all the various combinations which theſe letters would admit of. And conſequently, if the world had already laſted 6000 years, it would require 9576907016586170 ſuch ages to make all theſe combinations, without ever ſtopping for one ſingle ſecond of time.

Suppoſing a ſquare Ciſtern to be a Mile wide and a Mile deep, or to contain a Cubic Mile of Water; and that a Cubic Yard of Water ſhould run off from it every Minute until it was quite emptied. Qu. How much Time would all the Water take to run out of the Ciſtern?

Anſ. 5451776000 minutes (for ſo many cubic yards there are in a cubic mile) or 10365 Julian years 139 days 7 hours 20 minutes.

Concerning the Strength of Steam. From the Reverend Mr. Mitchell's *Treatiſe on Earthquakes.*

" There are many effects produced by the vapour of water, when intenſely heated; which make it probable that the force of gunpowder is not near equal to it. The effects of an exceeding ſmall quantity of water, upon which melted metals are accidentally poured are ſuch, I think, as could no ways be expected from the like quantity of gunpowder. Founders, if they are not careful, often experience theſe effects to their coſt.— An accident of this kind happened about forty years ago, at the caſting of two braſs cannon at Windmill-Hill, Morefields. The heat, ſays Cranmer, of the metal of the firſt gun drove ſo much damp into the mould of the ſecond, which was near it, that, as ſoon as the metal was let into it, it blew up with the greateſt violence, tearing up the ground ſome feet deep, breaking down the furnace, untiling the houſe, killing many ſpectators on the ſpot with the ſteams of the melted metal, and ſcalding many others in a moſt miſerable manner.

Volcanos prove that there are fires within the earth, far below its ſurface; and over ſome of theſe fires there may be caverns of water. When any of the water finds its way through the bottom of ſuch a cavern, and falls down into the fire, the water will be immediately rarified into ſteam; and the elaſticity thereof will heave up the ground above it, and make an earthquake. The deeper the fire is, the further will the earthquake be extended."

MATHEMATICAL
TABLES
FOR
Dividing the LINES on SCALES and SECTORS.

In theſe Tables I have only numbered the whole degrees, the intermediate lines ſhewing how many parts each of them is divided into: as where there are three ſuch lines, they denote the degree to be divided into quarters; where two, into thirds; and where one, into halves.

Natural Chords.

Degrees.	*Chord.* Parts.	Degrees.	*Chord.* Parts.	Degrees.	*Chord.* Parts.	Degrees.	*Chord.* Parts.
¼	4.36	¼	143.86	¼	282.66	¼	420.09
	8.72		148.21		286.98		424.35
	13.09		152.56		291.30		428.61
1	17.45	9	156.91	17	295.61	25	432.87
	21.81		161.26		299.93		437.13
	26.17		165.61		304.24		441.39
	30.53		169.96		308.55		445.65
2	34.90	10	174.31	18	312.86	26	449.90
	39.26		178.66		317.17		454.15
	43.62		183.01		321.48		458.40
	47.98		187.35		325.79		462.65
3	52.35	11	191.69	19	330.09	27	466.89
	56.71		196.02		334.39		471.13
	61.07		200.37		338.69		475.37
	65.43		204.71		342.99		479.61
4	69.79	12	209.05	20	347.29	28	483.84
	74.15		213.39		351.59		488.07
	78.51		217.73		355.88		492.30
	82.87		222.07		360.18		496.53
5	87.23	13	226.40	21	364.47	29	500.76
	91.59		230.74		368.76		504.98
	95.95		235.07		373.04		509.20
	100.31		239.40		377.33		513.42
6	104.67	14	243.73	22	381.61	30	517.63
	109.03		248.06		385.89		521.85
	113.38		252.39		390.18		526.06
	117.74		256.72		394.46		530.27
7	122.09	15	261.05	23	398.73	31	534.47
	126.45		265.38		403.01		538.68
	130.80		269.70		407.28		542.88
	135.16		274.02		411.55		547.08
8	139.51	16	278.34	24	415.82	32	551.27

Natural Chords.

Degrees.	*Chord.* Parts.	Degrees.	*Chord.* Parts.	Degrees.	*Chord.* Parts.	Degrees.	*Chord.* Parts.
¼	555.46	¼	688.19	¼	817.45	¼	942.79
	559.65		692.23		821.43		946.63
	563.84		696.37		825.41		950.47
33	568.03	41	700.41	49	829.38	57	954.31
	572.21		704.55		833.35		958.14
	576.39		708.58		837.31		961.97
	580.56		712.71		841.27		965.79
34	584.74	42	716.73	50	845.23	58	969.61
	588.92		720.81		849.18		973.43
	593.09		724.87		853.13		977.24
	597.25		728.94		857.08		981.04
35	601.41	43	733.00	51	861.02	59	984.84
	605.57		737.06		864.96		987.64
	609.72		741.11		868.89		992.43
	613.88		745.16		872.82		996.22
36	618.03	44	749.21	52	876.74	60	1000.00
	622.18		753.25		880.66		1003.77
	626.32		757.29		884.57		1007.54
	630.46		761.33		888.48		1011.31
37	634.60	45	765.36	53	892.39	61	1015.07
	638.74		769.39		896.29		1018.83
	642.87		773.42		900.19		1022.58
	647.00		777.44		904.09		1026.33
38	651.13	46	781.46	54	907.98	62	1030.07
	655.26		785.47		911.85		1033.81
	659.38		789.48		915.74		1037.54
	663.50		793.49		919.62		1041.27
39	667.61	47	797.49	55	923.49	63	1044.99
	671.72		801.49		927.36		1048.71
	675.83		805.49		931.22		1052.42
	679.94		809.48		935.08		1056.13
40	684.04	48	813.47	56	938.94	64	1059.83

Natural Chords.

Degrees.	*Chord.* Parts.	Degrees.	*Chord.* Parts.	Degrees.	*Chord.* Parts.	Degrees.	*Chord.* Parts.
¼	1063.52	¼	1164.95	¼	1262.03	¼	1354.39
	1067.22		1168.49		1265.41		1357.60
	1070.91		1172.03		1268.78		1360.80
65	1074.59	72	1175.57	79	1272.15	86	1363.99
	1078.27		1179.09		1275.51		1367.18
	1081.94		1182.61		1278.87		1370.36
	1085.61		1186.13		1282.22		1373.53
66	1089.27	73	1189.64	80	1285.57	87	1376.70
	1092.93		1193.14		1288.91		1379.86
	1096.58		1196.64		1292.24		1383.02
	1100.23		1200.14		1295.57		1386.17
67	1103.87	74	1203.63	81	1298.89	88	1389.31
	1107.51		1207.11		1302.20		1392.45
	1111.14		1210.58		1305.51		1395.58
	1114.76		1214.05		1308.81		1398.70
68	1118.38	75	1217.52	82	1312.11	89	1401.81
	1121.99		1220.98		1315.40		1404.92
	1125.60		1224.43		1318.69		1408.02
	1128.21		1227.88		1321.97		1411.12
69	1132.81	76	1231.32	83	1325.24	90	1414.21
	1136.40		1234.75		1328.50		
	1139.99		1238.18		1331.76		
	1143.57		1241.60		1335.01		
70	1147.15	77	1245.02	84	1338.26		
	1150.72		1248.43		1341.50		
	1154.29		1251.84		1344.73		
	1157.85		1255.24		1347.96		
71	1161.40	78	1258.64	85	1351.18		

End of the Table of Natural Chords.

Natural Sines.

Degrees.	*Sine.* Parts.	Degrees.	*Sine.* Parts.	Degrees.	*Sine.* Parts.	Degrees.	*Sine.* Parts.
¼	4.36	¼	143.49	¼	279.83	¼	410.72
	8.72		147.81		284.02		414.69
	13.09		152.12		288.20		418.66
1	17.45	9	156.43	17	292.37	25	422.62
	21.81		160.74		296.54		426.57
	26.17		165.05		300.71		430.51
	30.53		169.35		304.86		434.45
2	34.90	10	173.65	18	309.02	26	438.37
	39.26		177.94		313.16		442.29
	43.62		182.23		317.30		446.20
	47.98		186.52		321.44		450.10
3	52.34	11	190.81	19	325.57	27	453.99
	56.69		195.09		329.69		457.87
	61.05		199.37		333.81		461.75
	65.40		203.64		337.92		465.61
4	69.76	12	207.91	20	342.02	28	469.47
	74.11		212.18		346.12		473.32
	78.46		216.44		350.21		477.16
	82.81		220.70		354.29		480.99
5	87.16	13	224.95	21	358.37	29	484.81
	91.50		229.20		362.44		488.62
	95.85		233.45		366.50		492.42
	100.19		237.69		370.56		496.21
6	104.53	14	241.92	22	374.61	30	500.00
	108.87		246.15		378.65		503.77
	113.20		250.38		382.68		507.54
	117.54		254.60		386.71		511.29
7	121.87	15	258.82	23	390.73	31	515.04
	126.20		263.03		394.74		518.77
	130.53		267.24		398.75		522.50
	134.85		271.44		402.75		526.21
8	139.17	16	275.64	24	406.74	32	529.92

Natural Sines.

Degrees.	*Sine.* Parts.	Degrees.	*Sine.* Parts.	Degrees.	*Sine.* Parts.	Degrees.	*Sine.* Parts.
¼	533.61	¼	646.12	¼	746.06	¼	831.47
	537.30		649.49		748.96		833.86
	540.97		652.76		751.84		836.29
33	544.64	41	656.06	49	754.71	57	838.67
	548.29		659.35		757.56		841.04
	551.94		662.62		760.41		843.39
	555.57		665.88		763.23		845.73
34	559.19	42	669.13	50	766.04	58	848.05
	562.80		672.37		768.84		850.35
	566.41		675.59		771.62		852.64
	569.99		678.80		774.39		854.91
35	573.58	43	682.00	51	777.15	59	857.17
	577.14		685.18		779.88		859.41
	580.70		688.35		782.61		861.63
	584.25		691.51		785.32		863.84
36	587.79	44	694.66	52	788.01	60	866.03
	591.31		697.79		790.69		868.20
	594.82		700.91		793.35		870.36
	598.32		704.01		796.00		872.50
37	601.81	45	707.11	53	798.64	61	874.62
	605.29		710.19		801.25		876.73
	608.76		713.25		803.86		878.82
	612.22		716.30		806.44		880.89
38	615.66	46	719.34	54	809.02	62	882.95
	619.09		722.36		811.57		884.99
	622.51		725.37		814.12		887.01
	625.92		728.37		816.64		889.02
39	629.32	47	731.35	55	819.15	63	891.01
	632.71		734.32		821.65		892.98
	636.08		737.28		824.13		894.93
	639.44		740.22		826.59		896.87
40	642.79	48	743.14	56	829.04	64	898.79

Natural Sines.

Degrees.	*Sine.* Parts.	Degrees.	*Sine.* Parts.
¼	900.70	½	963.63
	902.59	75	965.93
	904.46	76	970.30
65	906.31	77	974.37
	908.14	78	978.15
	909.96	79	981.63
	911.76	80	984.81
66	913.55	81	987.69
	915.31	82	990.27
	917.06	83	992.55
	918.79	84	994.52
67	920.50	85	996.19
	922.20	90	1000.00
	923.88		
	925.54		
68	927.18		
	928.81		
	930.42		
	932.01		
69	933.58		
	935.14		
	936.67		
	938.19		
70	939.69		
	942.64		
71	945.52		
	948.32		
72	951.06		
	953.72		
73	956.30		
	958.82		
74	961.26		

End of the Table of Natural Sines.

The Secants begin where the Sines end.

As the ſpaces which contain the degrees of the Sines are ſo very ſmall when they go beyond 70, that they could not be divided into quarters without hurting the eye, even if the whole line of ſines was a foot long, nor into any thing leſs than halves between 70 and 75, nor in leſs than whole degrees from 75 to 85, nor into any degrees at all between 85 and 90; it was thought needleſs to put any more numbers into this Table than what are ſufficient to anſwer the purpoſe for which it was intended.

Natural Tangents.

Degrees.	*Tang.* Parts.	Degrees.	*Tang.* Parts.	Degrees.	*Tang.* Parts.	Degrees.	*Tang.* Parts.
¼	4.36	¼	144.99	¼	291.47	¼	450.47
	8.73		149.45		296.21		455.73
	13.09		153.91		300.97		461.01
1	17.46	9	158.38	17	305.73	25	466.31
	21.82		162.86		310.51		471.63
	26.19		167.34		315.30		476.98
	30.55		171.83		320.10		482.34
2	34.92	10	176.33	18	324.92	26	487.73
	39.29		180.83		329.75		493.15
	43.66		185.34		334.60		498.58
	48.03		189.86		339.45		504.04
3	52.41	11	194.38	19	344.33	27	509.53
	56.78		198.91		349.22		515.03
	61.16		263.45		354.12		520.57
	65.54		208.00		359.03		526.13
4	69.93	12	212.56	20	363.97	28	531.71
	74.31		217.12		368.92		537.32
	78.70		221.69		373.88		542.96
	83.09		226.26		378.87		548.62
5	87.49	13	230.87	21	383.86	29	554.31
	91.89		235.47		388.88		560.03
	96.29		240.08		393.91		565.77
	100.69		244.70		398.96		571.55
6	105.10	14	249.33	22	404.03	30	577.35
	109.52		253.97		409.11		583.18
	113.93		258.62		414.21		589.04
	118.36		263.28		419.33		594.94
7	122.78	15	267.95	23	424.47	31	600.86
	127.22		272.63		429.63		606.81
	131.65		277.32		434.81		612.80
	136.09		282.03		440.01		618.82
8	140.54	16	286.74	24	445.23	32	621.87

Natural Tangents.

Degrees.	*Tang.* Parts.	Degrees.	*Tang.* Parts.	Degrees.	*Tang.* Parts.	Degrees.	*Tang.* Parts.
¼	630 95	¼	846.56	¼	1120.41	¼	1496.61
	637.07		854.08		1130.29		1510.84
	643.22		861.66		1140.28		1525.25
33	649.41	41	869.29	49	1150.37	57	1539.86
	655.63		876.98		1160.56		1554.67
	661.89		884.73		1170.85		1569.69
	668.18		892.53		1181.25		1584.90
34	674.51	42	900.40	50	1191.75	58	1600.33
	680.88		908.34		1202.37		1615.98
	687.28		916.33		1213.10		1631.85
	693.72		924.39		1223.94		1647.95
35	700.21	43	932.52	51	1234.90	59	1664.28
	706.73		940.71		1245.97		1680.85
	713.29		948.96		1257.17		1697.66
	719.90		957.29		1268.49		1714.73
36	726.54	44	965 69	52	1279.94	60	1732.05
	733.23		974.16		1291.52		1749 64
	739.96		982.70		1303.23		1767.49
	746.74		991.31		1315.07		1785.63
37	753.55	45	1000.00	53	1327.04	61	1804.05
	760.42		1008.76		1339.16		1822.76
	767.33		1017.61		1351.42		1841.77
	774.28		1026.53		1363.83		1861.09
38	781.29	46	1035.53	54	1376.38	62	1880.73
	788.34		1044.61		1389.09		1900.69
	795.44		1053.78		1401.95		1920.98
	802.58		1063.03		1414.97		1941.62
39	809.78	47	1072.37	55	1428.15	63	1962.61
	817.03		1081.79		1441.49		1983.96
	824.34		1091.31		1455.01		2005.69
	831.69		1100.91		1468.70		2027.80
40	839.10	48	1110.61	56	1482.56	64	2050.30

Natural Tangents.

Degrees.	*Tang.* Parts.	Degrees.	*Tang.* Parts.	Degrees.	*Tang* Parts.	Degrees.	*Tang.* Parts.
¼	2073.21	¼	2506.52	¼	3124.00	¼	4086.66
	2096.54		2538.65		3171.59		4165.30
	2120.30		2571.50		3220 53		4246.85
65	2144.51	69	2605.09	73	3270.85	77	4331.48
	2169.17		2639.45		3322.64		4419.36
	2194.30		2674.62		3375.94		4510.71
	2219.92		2710.62		3430.84		4605.72
66	2246.04	70	2747.48	74	3487.41	78	4704.63
	2272.67		2785.23		3545.73		4807.69
	2299.84		2823.91		3605.88		4915.16
	2327.56		2863.56		3667.96		5027.34
67	2355.85	71	2904.21	75	3732.05	79	5144.55
	2384.73		2945.90		3798.27		5267.15
	2414.21		2988.68		3866.71		5395.52
	2444.33		3032.60		3937.51		5530.07
68	2475.09	72	3077.68	76	4010.78	80	5671.28

As the Tangents are never laid down further than to 80 degrees on common ſcales, it would be needleſs to carry them further in this Table.

The ſemi-tangents may be laid down on a ſcale by taking out the tangents of half the number of degrees in this Table.—Thus, the ſemi-tangent of a whole degree is the whole tangent of half a degree: the ſemi-tangent of 2 degrees is the whole tangent of 1 degree: the ſemi-tangent of 3 degrees is the whole tangent of 1½ degree: the ſemi-tangent of 4 degrees is the whole tangent of 2 degrees: and ſo on. They are never ſubdivided further than to half degrees.

Natural Secants.

Degrees.	*Sec.* Parts.	Degrees.	*Sec.* Parts.	Degrees.	*Sec.* Parts.	Degrees.	*Sec.* Parts.
0	1000.00	½	1260.47	¾	1459.46	¾	1732.67
10	1015.43	38	1269.02	47	1466.28	55	1743.45
15	1035.28		1277.78		1473.19		1754.40
16	1040.30	39	1286.76		1480.19		1765.52
17	1045.69		1295.97		1487.28		1776.81
18	1051.46	40	1305.41	48	1494.48	56	1788.29
19	1057.62		1310.22		1501.77		1799.95
20	1064.18		1315.09		1509.16		1811.80
21	1071.14		1320.02		1516.65		1823.84
22	1078.53	41	1325.01	49	1524.25	57	1836.08
23	1086.30		1330.07		1531.96		1848.51
24	1094.64		1335.19		1539.79		1861.16
25	1103.38		1340.38		1547.69		1874.01
26	1112.60	42	1345.63	50	1555.72	58	1887.08
27	1122.33		1350.95		1563.87		1900.37
28	1132.57		1356.34		1572.13		1913.88
29	1143.35		1361.80		1580.51		1927.62
30	1154.70	43	1367.33	51	1589.02	59	1941.60
	1160.59		1372.93		1597.64		1955.82
31	1166.63		1378.56		1606.39		1970.29
	1172.83		1384.34		1615.26		1985.02
32	1179.18	44	1390.16	52	1624.27	60	2000.00
	1185.69		1396.06		1633.41		2015.25
33	1192.36		1402.03		1642.68		2030.77
	1199.20		1408.08	—	1652.09		2046.57
34	1206.22	45	1414.21	53	1661.64	61	2062.67
	1213.41		1420.42		1671.33		2079.05
35	1220.77		1426.72		1681.17		2095.74
	1228.33		1433.09		1691.16		2112.74
36	1236.07	46	1439.56	54	1701.30	62	2130.05
	1244.00		1446.10		1711.60		2147.70
37	1252.14		1452.74		1722.05		2165.68

Natural Secants.

Deg.	*Sec.* Parts.	Deg.	*Sec.* Parts.	Deg.	*Sec.* Parts.	Deg.	*Sec.* Parts.
¾	2184.01	¼	2585.91	¾	3193.22	¼	4207.23
63	2202.69		2613.13	72	3236.07		4283.66
	2221.74		2640.97		3280.15		4362.99
	2241.16	68	2669.47		3325.51	77	4445.41
	2260.97		2698.64		3372.21		4531.09
64	2281.17		2728.50	73	3420.30		4620.22
	2301.79		2759.09		3469.86		4713.03
	2322.82	69	2790.43		3520 94	78	4809.73
	2344.29		2822.54		3573.61		4910.58
65	2366.20		2855.45	74	3627.96		5015.85
	2388.57		2889.20		3684.05		5125.83
	2411.42	70	2923.80		3741.98	79	5248.84
	2434.76		2959.31		3801.83		5361.23
66	2458.49		2995.74	75	3863.70		5487.40
	2482.95		3033.15		3927.71		5619.76
	2507.84	71	3071.55		3993.93	80	5758.77
	2533.29		3111.01		4062.51		The End.
67	2559.30		3151.55	76	4133.57		

Natural Rhumbs.

R.	Parts.	R.	Parts.	R.	Parts.	R.	Parts.
¼	49.09	¼	438.19	¼	810.48	¼	1151.62
	98.14		485.96		855.10		1191.40
	147.12		533.42		899.22		1230.46
1	196.02	3	580.56	5	942.79	7	1268.78
	224.82		627.36		985.82		1306.34
	293.46		673.78		1028.20		1343.12
	341.92		719.80		1069.98		1379.07
2	390.18	4	765.36	6	1111.14	8	1414.21

This and all the preceding Tables are fitted to one and the ſame Radius on the plain ſcale.

A TABLE *ſhewing in what Parallel of Latitude any given Number of Geographical Miles make a Degree of Longitude, and their Projection on a plain Scale.*

Miles.	Latitude.		Nat. Chord. Parts.	Miles.	Latitude.		Nat. Chord. Parts.
	°	′			°	′	
60	00	00	00.00	30	60	0	1000.00
59	10	28½	182.86	29	61	6	1016.58
58	14	50	258 17	28	62	11	1032.82
57	18	11⅔	316.22	27	63	15⅓	1048.79
56	21	2⅓	365.43	26	64	19¼	1064 60
55	23	33½	408.28	25	65	22½	1080.11
54	25	50½	447.49	24	66	25⅓	1095.45
53	27	57	483.00	23	67	27½	1110.53
52	29	55⅔	516.42	22	68	29½	1125.49
51	31	47¼	547.71	21	69	30⅔	1140.15
50	33	33½	577.37	20	70	31⅔	1154.79
49	35	15	605.57	19	71	32⅓	1168.99
48	36	52	632.40	18	72	32½	1183.21
47	38	26	658.28	17	73	32⅓	1197.20
46	39	56⅔	683.12	16	74	32	1211.05
45	41	24½	707.08	15	75	31	1224.66
44	42	50	730.30	14	76	30⅓	1238.27
43	44	13½	752.84	13	77	29⅓	1251.70
42	45	34⅓	774.58	12	78	27⅔	1264.87
41	46	52¾	795.82	11	79	26	1277.98
40	48	11½	816.52	10	80	24⅔	1291.07
39	49	27½	836.65	9	81	22⅓	1303.82
38	50	42¼	856.36	8	82	20⅓	1316.58
37	51	55⅓	875.52	7	83	18	1329.16
36	53	8	894.48	6	84	15	1341.50
35	54	19	912.90	5	85	13	1353.97
34	55	29	930.97	4	86	10⅔	1366.26
33	56	38	948.69	3	87	8	1378.40
32	57	46	966.06	2	88	5½	1390.47
31	58	53½	983.19	1	89	2⅔	1402.36
30	60	0	1000.00	0	90	0	1414.21

Latitudes, for the Dialing Scale.

Deg.	Parts.	Deg.	Parts.	Deg.	Parts.
1	24.7	31	647.5	61	931.1
2	49.3	32	662.2	62	936.0
3	73.9	33	676.4	63	940.8
4	98.4	34	690.2	64	945.4
5	122.8	35	703.6	65	949.6
6	147.0	36	716.6	66	953.9
7	171.1	37	729.2	67	957.8
8	194.9	38	741.4	68	961.5
9	218.6	39	753.2	69	965.1
10	241.9	40	764.7	70	968.5
11	265.0	41	775.8	71	971.6
12	287.9	42	786.5	72	974.5
13	310.4	43	796.8	73	977.4
14	332.5	44	806.7	74	980.1
15	354.3	45	816.5	75	982.5
16	375.8	46	825.9	76	984.8
17	396.9	47	834.8	77	986.9
18	417.6	48	843.6	78	988.8
19	437.8	49	851.9	79	990.6
20	457.7	50	860.0	80	992.4
21	477.3	51	867.8	85	998.2
22	496.1	52	875.3	90	1000.0
23	514.6	53	882.5	No Degrees can be put into this ſcale between 80 and 85, nor between 85 and 90.	
24	532.8	54	889.5		
25	550.5	55	896.2		
26	567.8	56	902.6		
27	584.6	57	908.8		
28	601.0	58	914.7		
29	616.9	59	920.3		
30	632.5	60	925.8		

Hours adapted to the preceding Table of Latitudes, for Dialing.

Hours and Minutes.		Parts.	Hours and Minutes.		Parts.
XII	0	0.0	III	0	707.1
	5	30.3		5	722.5
	10	59.1		10	737.9
	15	87.0		15	753.4
	20	113.8		20	769.0
	25	139.6		25	784.6
	30	164.5		30	800.2
	35	188.8		35	816.0
	40	212.0		40	831.9
	45	234.6		45	847.8
	50	256.7		50	863.8
	55	278.0		55	880.1
I	0	298.8	IIII	0	896.5
	5	319.2		5	913.1
	10	339.0		10	930.0
	15	358.4		15	947.1
	20	377.3		20	964.5
	25	395.9		25	982.0
	30	414.2		30	1000.0
	35	432.2		35	1018.3
	40	449.7		40	1036.9
	45	467.1		45	1055.8
	50	484.2		50	1075.2
	55	501.1		55	1095.0
II	0	517.7	V	0	1115.4
	5	534.1		5	1136.2
	10	550.4		10	1157.5
	15	566.4		15	1179.6
	20	582.4		20	1202.2
	25	598.3		25	1225.4
	30	614.0		30	1249.7
	35	629.6		35	1274.6
	40	645.2		40	1300.4
	45	660.8		45	1327.2
	50	676.3		50	1355.1
	55	691.7		55	1383.9
III	0	707.1	VI	0	1414.2

Inclination of Meridians.

Deg.	Parts.	Deg.	Parts.	Deg.	Parts.
1	24.3	31	530.8	61	909.8
2	47.7	32	543.8	62	923.3
3	70.4	33	556.8	63	936.9
4	92.4	34	569.7	64	950.6
5	113.8	35	582.4	65	964.5
6	134.5	36	595.1	66	978.5
7	154.6	37	607.7	67	992.8
8	174.2	38	620.3	68	1007.3
9	193.4	39	632.8	69	1022.0
10	212.0	40	645.2	70	1036.9
11	230.1	41	657.7	71	1052.0
12	247.9	42	670.0	72	1067.4
13	265.3	43	682.4	73	1083.1
14	282.2	44	694.7	74	1099.1
15	298.8	45	707.1	75	1115.4
16	315.1	46	719.5	76	1132.0
17	331.1	47	731.8	77	1148.9
18	346.8	48	744.2	78	1166.3
19	362.2	49	756.5	79	1184.1
20	377.3	50	769.0	80	1202.2
21	392.2	51	781.4	81	1220.8
22	406.9	52	793.9	82	1240.0
23	421.4	53	806.5	83	1259.6
24	435.7	54	819.1	84	1279.7
25	449.7	55	831.8	85	1300.4
26	463.6	56	844.5	86	1321.8
27	477.3	57	857.4	87	1343.8
28	490.9	58	870.4	88	1366.5
29	504.4	59	883.4	89	1389.9
30	517.7	60	896.5	90	1414.2

Adapted to the preceding Line of Hours, and Line of Latitudes.

The preceding Tables are for graduating the Lines of Natural Chords, Sines, Tangents, and Secants, &c. on common plain Scales and Sectors.—The following Tables are for laying down Gunter's logarithmic Lines of Numbers, Sines, Tangents, Verſed Sines, Meridional Parts, &c. on his Scales.

Numbers. The Logarithm of 1 = 0.

N.	Parts.	N.	Parts.	N.	Parts.
1.01	4.32	1.31	117.27	1.61	206.83
1.02	8.60	1.32	120.57	1.62	209.52
1.03	12.84	1.33	123.85	1.63	212.19
1.04	17.03	1.34	127.10	1.64	214.84
1.05	21.19	1.35	130.33	1.65	217.48
1.06	25.31	1.36	133.54	1.66	220.11
1.07	29.38	1.37	136.72	1.67	222.72
1.08	33.42	1.38	139.88	1.68	225.31
1.09	37.43	1.39	143.01	1.69	227.89
1.10	41.39	1.40	146.13	1.70	230.45
1.11	45.32	1.41	149.22	1.71	233.00
1.12	49.22	1.42	152.29	1.72	235.53
1.13	53.08	1.43	155.37	1.73	238.05
1.14	56.90	1.44	158.36	1.74	240.55
1.15	60.70	1.45	161.37	1.75	243.04
1.16	64.46	1.46	164.35	1.76	245.51
1.17	68.19	1.47	167.32	1.77	247.97
1.18	71.88	1.48	170.26	1.78	250.42
1.19	75.55	1.49	173.19	1.79	252.85
1.20	79.18	1.50	176.09	1.80	255.27
1.21	82.79	1.51	178.98	1.81	257.68
1.22	86.36	1.52	181.84	1.82	260.07
1.23	89.91	1.53	184.69	1.83	262.45
1.24	93.42	1.54	187.52	1.84	264.82
1.25	96.91	1.55	190.33	1.85	267.17
1.26	100.37	1.56	193.12	1.86	269.51
1.27	103.80	1.57	195.90	1.87	271.84
1.28	107.21	1.58	198.66	1.88	274.16
1.29	110.59	1.59	201.34	1.89	276.46
1.30	113.94	1.60	204.12	1.90	278.75

Gunter's Line of Numbers.

N.	Parts.	N.	Parts.	N.	Parts.
1.91	281.03	2.42	383.82	3.05	484.30
1.92	283.30	2.44	387.39	3.10	491.36
1.93	285.56	2.46	390.94	3.15	498.31
1.94	287.80	2.48	394.45	3.20	505.15
1.95	290.03	2.50	397.94	3.25	511.88
1.96	292.26	2.52	401.40	3.30	518.51
1.97	294.47	2.54	404.83	3.35	525.04
1.98	296.66	2.56	408.24	3.40	531.48
1.99	298.85	2.58	411.62	3.45	537.82
2.00	301.03	2.60	414.97	3.50	544.07
2.02	305.35	2.62	418.30	3.55	550.23
2.04	309.63	2.64	421.60	3.60	556.30
2.06	313.87	2.66	424.88	3.65	562.29
2.08	318.06	2.68	428.13	3.70	568.20
2.10	322.22	2.70	431.36	3.75	574.03
2.12	326.34	2.72	434.57	3.80	579.78
2.14	330.41	2.74	437.75	3.85	585.46
2.16	334.45	2.76	440.91	3.90	591.06
2.18	338.46	2.78	444.04	3.95	596.60
2.20	342.42	2.80	447.16	4.00	602.06
2.22	346.35	2.82	450.25	4.05	607.45
2.24	350.25	2.84	453.32	4.10	612.78
2.26	354.11	2.86	456.37	4.15	618.05
2.28	357.93	2.88	459.39	4.20	623.25
2.30	361.73	2.90	462.40	4.25	628.40
2.32	365.49	2.92	465.38	4.30	633.47
2.34	369.22	2.94	468.35	4.35	638.49
2.36	372.91	2.96	471.29	4.40	643.49
2.38	376.58	2.98	474.22	4.45	648.36
2.40	380.21	3.00	477.12	4.50	653.21

Gunter's Line of Numbers.

N.	Parts.	N.	Parts.	N.	Parts.
4.55	658.01	6.05	781.75	7.55	877.95
4.60	662.76	6.10	785.33	7.60	880.81
4.65	667.45	6.15	788.75	7.65	883.66
4.70	672.10	6.20	792.39	7.70	886.49
4.75	676.69	6.25	795.88	7.75	889.30
4.80	681.24	6.30	799.34	7.80	892.09
4.85	685.74	6.35	802.77	7.85	894.87
4.90	690.20	6.40	806.18	7.90	897.63
4.95	694.61	6.45	809.56	7.95	900.37
5.00	698.97	6.50	812.91	8.00	903.09
5.05	703.29	6.55	816.24	8.05	905.71
5.10	707.57	6.60	819.54	8.10	908.48
5.15	711.81	6.65	822.82	8.15	911.16
5.20	716.00	6.70	826.07	8.20	913.81
5.25	720.16	6.75	829.30	8.25	916.45
5.30	724.28	6.80	832.51	8.30	919.08
5.35	728.35	6.85	835.69	8.35	921.69
5.40	732.39	6.90	838.84	8.40	924.28
5.45	736.40	6.95	841.98	8.45	926.86
5.50	740.36	7.00	845.10	8.50	929.42
5.55	744.29	7.05	848.19	8.55	931.97
5.60	748.19	7.10	851.26	8.60	934.50
5.65	752.05	7.15	854.30	8.65	937.02
5.70	755.87	7.20	857.33	8.70	939.52
5.75	759.67	7.25	860.34	8.75	942.01
5.80	763.43	7.30	863.32	8.80	944.48
5.85	767.16	7.35	866.29	8.85	946.94
5.90	770.85	7.40	869.23	8.90	949.39
5.95	774.52	7.45	872.16	8.95	951.82
6.00	778.15	7.50	875.06	9.00	954.24

Gunter's Line of Numbers.

N.	Parts.	N.	Parts.	N.	Parts.
9.05	956.65	9.40	973.13	9.75	989.00
9.10	959.04	9.45	975.43	9.80	991.23
9.15	961.42	9.50	977.72	9.85	993.44
9.20	963.79	9.55	980.00	9.90	995.63
9.25	966.14	9.60	982.27	9.95	997.82
9.30	968.48	9.65	984.53	10.00	1000.00
9.35	970.81	9.70	986.77	The End.	

All the number of parts into which the double Line of Numbers (from 1 to 10) on a ſcale two feet long can well be divided, are inſerted in this Table.

The Line of Numbers is marked with the numeral figures 1, 2, 3, 4, 5, 6, 7, 8, 9, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10.—The ſpace between 1 and 2 may be divided into 100 parts; from 2 to 3, into 50; but all the others into no more than 20 each, without hurting the eye to look at ſuch ſmall diviſions.

But, in common practice, the ſpaces from the firſt 1 to the ſecond 1 are divided only into 10 parts each; and after that, from 1 to 2, into 50; from 2 to 5, into 20 each; and from 5 to 10, each ſpace is divided into no more than 10 parts; leaving the intermediate ſubdiviſions to be eſtimated by the eye, which may be eaſily done.

In the following Tables, D. ſtands for Degrees, and M. for Minutes of a Degree.

Gunter's Line of Sines.

D.	M.	Parts.	D.	M.	Parts.	D.	M.	Parts.
0	20	2235.25	3	15	1246.47	6	10	968.91
	25	2138.34		20	1235.49		15	963.10
	30	2059.16		25	1224.78		20	957.38
	35	1992.21		30	1214.32		25	951.72
	40	1934.22		35	1204.12		30	946.14
	45	1883 07		40	1194.15		35	940.63
	50	1837.32		45	1184.40		40	935.19
	55	1795.93		50	1174.87		45	929.82
1	0	1758.14		55	1165.54		50	924.52
	5	1723.39	4	0	1156.42		55	919.28
	10	1691.21		5	1147.48	7	0	914.11
	15	1661.25		10	1138.72		5	908.99
	20	1633.22		15	1130.13		10	903.94
	25	1606.90		20	1121.71		15	898.84
	30	1582.08		25	1115.10		20	894.01
	35	1558.61		30	1105.36		25	889.13
	40	1536.34		35	1097.40		30	884.30
	45	1515.15		40	1089.60		35	879.53
	50	1494.96		45	1081.92		40	874.81
	55	1475.66		50	1074.39		45	870.15
2	0	1457.18		55	1056.98		50	865.53
	5	1439.46	5	0	1059.70		55	860.96
	10	1422.43		5	1052.64	8	0	856.44
	15	1406.05		10	1045.50		5	851.97
	20	1390.27		15	1038.57		10	846.67
	25	1375.03		20	1031.75		15	843.17
	30	1360.32		25	1025.04		20	838.84
	35	1346.09		30	1018.43		25	834.55
	40	1332.31		35	1011.92		30	830.30
	45	1318.96		40	1005.50		35	826.09
	50	1306.00		45	999.18		40	821.93
	55	1293.42		50	992.96		45	817.80
3	0	1281.20		55	986.82		50	813.72
	5	1269.31	6	0	980.77		55	809.67
	10	1257.74		5	974.78	9	0	805.67

Gunter's Line of Sines.

D.	M.	Parts.	D.	M.	Parts.	D.	M.	Parts.
9	5	801.70	14	10	611.29	20	15	460.78
	10	797.77		20	606.31		30	455.67
	15	793.87		30	601.40		45	450.64
	20	790.01		40	596.54	21	0	445.67
	25	786.18		50	591.75		15	440.77
	30	782.39	15	0	587.00		30	435.92
	35	778.63		10	582.32		45	431.14
	40	774.91		20	577.68	22	0	426.42
	45	771.22		30	573.10		15	421.76
	50	767.56		40	568.57		30	417.16
	55	763.93		50	564.09		45	412.61
10	0	760.33	16	0	559.66	23	0	408.12
	10	753.23		10	555.28		15	403.68
	20	746.24		20	550.95		30	399.30
	30	739.37		30	546.66		45	394.97
	40	732.61		40	542.42	24	0	390.69
	50	725.95		50	538.22		15	386.46
11	0	719.40	17	0	534.06		30	382.27
	10	712.95		10	529.95		45	378.14
	20	706.60		20	525.88	25	0	374.05
	30	700.34		30	521.86		15	370.01
	40	694.18		40	517.87		30	366.02
	50	688.11		50	513.93		45	362.06
12	0	682.12	18	0	510.02	26	0	358.16
	10	676.22		10	506.15		15	354.29
	20	670.40		20	502.32		30	350.47
	30	664.66		30	498.52		45	346.69
	40	659.00		40	494.77	27	0	342.95
	50	653.42		50	491.04		15	339.25
13	0	647.91	19	0	487.36		30	335.59
	10	642.48		10	483.71		45	331.97
	20	637.11		20	480.09	28	0	328.39
	30	631.81		30	476.50		15	324.85
	40	626.59		40	472.95		30	321.34
	50	621.42		50	469.43		45	317.87
14	0	616.32	20	0	465.95	29	0	314.43

Gunter's Line of Sines.

Deg.	Parts.	Deg.	Parts.	Deg.	Parts.
¼	311.02	¼	208.24	½	89.31
	307.66		205.85	55	86.64
	304.33		203.48		84.01
30	301.03	39	201.13	56	81.43
	297.76		198.80		78.89
	294.53		196.49	57	76.41
	291.33		194.20		73.97
31	288.16	40	191.93	58	71.58
	285.02		187.46		69.23
	281.92	41	183.06	59	66.93
	278.84		178.73		64.68
32	275.79	42	174.49	60	62.47
	272.77		170.32	61	58.18
	269.78	43	166.22	62	54.07
	266.82		162.19	63	50.01
33	263.89	44	158.23	64	46.34
	260.99		154.34	65	42.72
	258.11	45	150.51	66	39.27
	255.26		146.76	67	35.97
34	252.44	46	143.07	68	32.83
	249.64		139.44	69	29.85
	246.87	47	135.87	70	27.01
	244.13		132.37	71	24.33
35	241.41	48	128.93	72	21.79
	238.71		125.54	73	19.40
	236.05	49	122.22	74	17.16
	233.40		118.95	75	15.06
36	230.78	50	115.75	76	13.10
	228.18		112.59	77	11.28
	225.61	51	109.50	78	9.60
	223.06		106.46	79	8.05
37	220.54	52	103.47	80	6.65
	218.03		100.53	85	1.66
	215.55	53	97.65	90	0.00
	213.09		94.82	The End.	
38	210.66	54	92.04		

Each Degree from the beginning to the 10th may be divided into 12 parts; from 10 to 20 into 6; from 20 to 40 into 4; from 40 to 60 into 2; after that to 80 into whole Degrees; and then, between 80 and 90, only the 85th Degree can be put in.

Gunter's Line of Verſed Sines.

Deg	Parts.	Deg.	Parts.	Deg.	Parts.	Deg.	Parts.
0	0	60	124.93	93	324.37	111	493.74
10	3.31	61	129.36		328.38		499.28
12	4.77	62	133.87	94	332.43	112	504.88
14	6.49	63	138.47		336.41		510.52
16	8.16	64	143.16	95	340.63	113	516.22
18	10.76	65	147.94		344.79		521.97
20	13.30	66	152.81	96	348.98	114	527.78
22	16.10	67	157.79		353.20		533.65
24	19.19	68	162.85	97	357.47	115	539.57
26	22.55	69	168.01		361.77		545.54
28	26.19	70	173.27	98	366.11	116	551.58
30	30.11	71	178.63		370.49		557.67
32	34.31	72	184.08	99	374.91	117	563.83
34	38.81	73	189.64		379.37		570.04
36	43.58	74	195.30	100	383.86	118	576.32
38	48.66	75	201.06		388.41		582.66
40	54.03	76	206.93	101	392.98	119	589.06
41	56.82	77	212.91		397.60		595.53
42	59.70	78	218.99	102	402.26	120	602.06
43	62.64	79	225.19		406.95		608.66
44	65.67	80	231.49	103	411.70	121	615.32
45	68.77	81	237.90		416.48		622.06
46	71.95	82	244.44	104	421.31	122	628.85
47	75.20	83	251.09		426.19		635.73
48	78.54	84	257.85	105	431.10	123	642.67
49	81.95	85	264.73		436.07		649.69
50	85.45	86	271.74	106	441.07	124	656.78
51	89.02	87	278.87		446.13		663.95
52	92.68	88	286.13	107	451.22	125	671.19
53	96.41	89	293.51		456.37		678.51
54	100.23	90	301.03	108	461.56	126	685.91
55	104.14		304.83		466.80		693.38
56	108.13	91	308.67	109	472.09	127	700.95
57	112.20		312.55		477.43		708.58
58	116.36	92	316.45	110	482.82	128	716.32
59	120.61		320.40		488.26		724.13

Gunter's Line of Verſed Sines.

Deg.	Parts.	Deg.	Parts.	Deg.	Parts.	Deg.	Parts.
129	732.03	½	901.27	½	1106.2	½	1382.2
	740.02		906.29		1112.7		1391.4
130	748.10	139	911.34	148	1119.3	157	1400.6
	752.16		916.42		1125.9		1410.0
	756.27		921.55		1132.6		1419.5
	760.12		926.69		1139.4		1429.1
131	764.55	140	931.89	149	1146.2	158	1438.8
	768.52		937.12		1153.0		1448.6
	772.91		942.38		1160.0		1458.5
	776.57		947.67		1167.0		1468.5
132	781.37	141	953.00	150	1174.0	159	1478.7
	785.45		958.37		1181.1		1489.0
	789.93		963.78		1188.3		1499.4
	793.97		969.21		1195.5		1510.0
133	798.60	142	974.71	151	1202.8	160	1520.6
	803.33		980.23		1210.1		1531.4
	807.36		985.80		1217.5		1542.4
	811.18		991.39		1225.0		1553.5
134	816.24	143	997.04	152	1232.6	161	1564.7
	820.72		1002.4		1240.3		1576.1
	825.22		1008.5		1248.0		1587.7
	829.45		1014.2		1255.8		1599.4
135	834.32	144	1020.0	153	1263.6	162	1611.3
	838.91		1025.9		1271.5		1623.3
	843.52		1031.7		1279.5		1635.6
	848.17		1037.9		1287.6		1648.0
136	852.84	145	1043.7	154	1295.8	163	1660.6
	857.54		1049.7		1304.1		1673.4
	862.28		1055.8		1312.4		1686.3
	867.04		1061.9		1320.8		1699.6
137	871.84	146	1068.1	155	1329.2	164	1712.9
	876.67		1074.6		1337.9		1726.5
	881.53		1080.6		1346.6		1740.3
	886.40		1086.9		1355.3		1754.3
138	891.34	147	1093.3	156	1364.2	165	1768.6
	896.29		1099.7		1373.2		1783.1

Gunter's Line of Verſed Sines.

Deg.	Parts.	Deg.	Parts	Deg.	Parts.	Deg	Parts.
½	1797.9	½	1859.6	½	1926.2	½	1998.4
	1812.9		1875.8		1943.7		2017.4
166	1828.2	167	1892.3	168	1961.5	The End.	
	1843.8		1909.1		1979.7		

Gunter's Line of Tangents.

D.	M.	Parts.	D.	M.	Parts.	D.	M.	Parts.
0	30	2059.14	2	45	1318.46	5	0	1058.05
	35	1992.19		50	1305.47		5	1050.83
	40	1934.19		55	1292.86		10	1043.73
	45	1883.04	3	0	1280.60		15	1036.75
	50	1837.27		5	1268.68		20	1029.87
	55	1795.87		10	1257.08		25	1023.09
1	0	1758.08		15	1245.77		30	1016.42
	5	1723.31		20	1234.75		35	1009.85
	10	1691.12		25	1224.00		40	1003.38
	15	1661.14		30	1213.51		45	996.99
	20	1633.11		35	1203.27		50	990.70
	25	1606.77		40	1193.26		55	984.50
	30	1581.93		45	1183.47	6	0	978.38
	35	1558.43		50	1173.90		5	972.34
	40	1536.15		55	1164.53		10	966.39
	45	1514.95	4	0	1155.36		15	960.52
	50	1494.73		5	1146.37		20	954.72
	55	1475.41		10	1137.57		25	948.99
2	0	1456.92		15	1128.94		30	943.34
	5	1439.17		20	1120.47		35	937.76
	10	1422.12		25	1112.16		40	932.25
	15	1405.72		30	1104.02		45	926.80
	20	1389.91		35	1096.01		50	921.42
	25	1374.65		40	1088.15		55	916.11
	30	1359.91		45	1080.43	7	0	910.86
	35	1345.65		50	1072.84		5	905.66
	40	1331.84		55	1065.38		10	900.53

Gunter's Line of Tangents.

D.	M.	Parts.	D.	M.	Parts.	D.	M.	Parts.	D.	Parts.
7	15	895.40		30	732.20		30	528.40	¾	350.54
	20	890.44		40	725.04		40	523.78	24	351.42
	25	885 48		50	718.14		50	519.20		346.34
	30	880.57	11	0	711.35	17	0	514.66		341.30
	35	875.72		10	705.32		10	510.16		336.29
	40	870.91		20	698.05		20	505.70	25	331.33
	45	866.16		30	691.54		30	501.28		326.40
	50	861.46		40	685.11		40	496.89		321.50
	55	856.80		50	678.78		50	492.54		316.64
8	0	852.20	12	0	672.53	18	0	488.22	26	311.82
	5	847.64		10	666.35		10	483.94		307.03
	10	843.12		20	660.26		20	479.69		302.26
	15	838.65		30	654.24		30	475.48		297.53
	20	834.23		40	648.30		40	471.30	27	292.83
	25	829.84		50	642.43		50	467.15		288.16
	30	824.64	13	0	636 64	19	0	463.03		283.52
	35	821.20		10	630.91		10	458.94		278.91
	40	816.94		20	625.24		20	454.88	28	274.33
	45	812.72		30	619.65		30	450.85		269.77
	50	808.54		40	614.11		40	446.85		265.24
	55	804.39		50	608.64		50	442.88		260.73
9	0	800.29	14	0	603.23	20	0	438.93	29	256.25
	5	796.22		10	597.88			433.07		251.79
	10	792.18		20	592.58			427.26		247.36
	15	788.18		30	587.34			421.51		242.95
	20	784.22		40	582.16	21		415.82	30	238.56
	25	780.29		50	577.03			410.19		234.20
	30	776.39	15	0	571.95			404.60		229.85
	35	772.53		10	566.92			399.07		225.53
	40	768.70		20	561.94	22		393.59	31	221.23
	45	764.90		30	557.01			388.16		216.94
	50	761.13		40	552.13			382.78		212.68
	55	757.39		50	547.29			377.44		208.44
10	0	753 68	16	0	542.50	23		372.15	32	204.21
	10	746.35		10	537.76			366.90		200.00
	20	739.14		20	533.06			361.70		195.81

Gunter's Line of Tangents.

Deg.	Parts.	Deg.	Parts.	Deg.	Parts.	Deg.	Parts.
¾	191.64	36	138.74	¼	87.76	½	37.95
33	187.48		134.76		83.90		34.14
	183.34		130.79		80.04	43	30.34
	179.22		126.83	40	76.19		26.55
	175.11	37	122.89		72.34		22.75
34	171.01		118.95		68.50		18.96
	166.93		115.02		64.67	44	15.16
	162.87		111.00	41	60.84		11.37
	158.81	38	107.19		57.01		7.58
35	154.77		103.29		53.19		3.79
	150.75		99.39		49.38	45	0.00
	146.73		95.51	42	45.56	The End.	
	142.73	39	91.63		41.75		

On Gunter's Scale, the Tangents begin at 0 degrees 35 minutes, at the left hand, and are thence numbered on to 45 degrees at the other end of the Scale; and thence backward, at the ſame diviſions, to 89 degrees 25 minutes. For, as the Tangents above and below 45 degrees are equally diſtant from the Radius, every grand diviſion may be doubly numbered. So that 40 and 50, 60 and 30, 70 and 20, 80 and 10, 85 and 5, are placed at the ſame grand diviſions of the Tangent line.

N. B. The firſt three numbers (0 d. 20 m. 25 m. and 30 m.) in the preceding Table of Sines, are ſuperfluous, becauſe they cannot be brought into the Scale.

Gunter's Lines of Rhumbs.

Deg.	M.	Rhumbs.	Sines.	Rhumbs.	Tangents.
0	0	0	Infinite.	0 8	Infinite.
2	48¾	¼	1309.21		1308.68
5	37½		1008.70		1006.60
8	26¼		833.48		828.75
11	15	1	709.76	1 7	701.34
14	3¾		614.43		601.21
16	52½		537.18		518.06
19	41¼		472.51		446.35
22	30	2	417.16	2 6	382.78
25	18¾		369.01		325.17
28	7½		326.61		272.04
30	56¼		288.95		222.30
33	45	3	255.26	3 5	175.11
36	33¾		224.97		129.80
39	22½		197.64		85.83
42	11¼		172.92		41.71
45	0	4	150.51	4 4	0.00
47	48¾		130 21		
50	37½		111.81		
53	26¼		95.17		
56	15	5	80.15		
59	3¾		66.65		
61	52½		54.57		
64	41¼		43.84		
67	30	6	34.38		
70	18¾		26.16		
73	7½		19.11		
75	56¼		13.21		
78	45	7	8.43		
81	33¾		4.73		
84	22½		2.10		
87	11¼		0.52		
90	0	8	0.00		

The Tangents and Co-tangents being equally diſtant from Radius, they are both found on the ſame point of the line: and therefore each grand diviſion is doubly figured; as 1, 7, are put to ſignify both one point and ſeven points of the Compaſs; and ſo on.

Gunter's Line of Meridional Parts.

Deg.	Parts.	Deg.	Parts.	Deg.	Parts.	Deg.	Parts.
0	0.0	16	194.5	28	350.2	⅔	503.8
	6.0		200.8		354.7	39	508.9
1	12.0	17	207.1		359.3		514.1
	18.0		213.3	29	363.9		519.3
2	24.0	18	219.6		368.2	40	524.5
	30.0		225.8		373.1		529.7
3	36.0	19	232.3	30	377.7		535.0
	42.0		238.6		382.3	41	540.3
4	48.0	20	245.0		387.9		545.6
	54.1	⅓	249.3	31	391.6		550.9
5	60.1	⅔	253.5		396.2	42	556.3
	66.1	21	257.8		400.9		561.7
6	72.1		262.1	32	405.7		567.1
	78.2		266.4		410.4	43	572.6
7	84.2	22	270.7		415.1		578.1
	90.3		275.0	33	419.9		583.6
8	96.3		279.3		424.7	44	589.1
	102.4	23	283.7		429.5		594.7
9	108.4		288.1	34	434.3		600.3
	114.5		292.4		439.1	45	605.9
10	120.6	24	296.8		443.9		611.6
	126.7		301.1	35	448.8		617.3
11	132.8		305.5		453.7	46	623.1
	138.9	25	309.9		458.7		628.9
12	145.0		314.4	36	463.5		634.7
	151.2		318.8		468.5	47	640.5
13	157.4	26	323.3		473.5		646.4
	163.5		327.7	37	478.5		652.3
14	169.7		332.2		483.5	48	658.3
	175.9	27	336.7		488.5		664.3
15	182.1		341.1	38	493.6		670.3
	188.3		345.7		498.7	49	676.4

Gunter's Line of Meridional Parts.

Deg.	Parts.	Deg.	Parts.	Deg.	Parts.	Deg.	Parts.
⅓	682.5	60	905.5	68	1126.1	76	1442.0
	688.7	¼	911.5		1134.2		1454.5
50	694.9		917.5		1142.3		1467.2
	701.1		923.7		1150.5		1480.2
	707.4	61	929.8	69	1158.9	77	1493.4
51	713.7		936.0		1167.3		1506.9
	720.1		942.3		1175.8		1520.6
	726.5		948.6		1184.4		1534.6
52	733.0	62	954.9	70	1193.1	78	1548.9
	739.5		961.4		1202.0		1563.5
	746.1		967.9		1210.9		1579.3
53	752.7		974.4		1219.9		1593.5
	759.4	63	980.9	71	1229 1	79	1609.0
	766.1		987.6		1238.4		1625.0
54	772.9		994.3		1247.8		1641.3
	779.7		1001.1		1257.3		1657.9
	786.6	64	1007.9	72	1266.9	80	1675.0
55	793.5		1014.7		1276.7		1692.5
	800.5		1021.6		1286.6		1710.4
	807.6		1028.7		1296.6		1728.8
56	814.7	65	1035.7	73	1306.8	81	1747.8
	821.9		1042.8		1317.2		1767.2
	829.2		1050.1		1327.7		1787.2
57	836.5		1057.3		1338.3		1807.8
	849.9	66	1065.7	74	1349.1	82	1829.0
	851.3		1072.1		1360.1		1850.9
58	858.8		1079.6		1371.2		1873.6
	866.4		1087.1		1382.5		1896.9
	874.1	67	1094.8	75	1394.0	83	1921.1
59	881.8		1102.5		1405.7		1946.2
	889.6		1110.2		1417.6		1972.2
	897.5		1118.1		1429.7		1999.2

Of the Conſtruction of the plain Scale, Sector, and Gunter's *Scale.*

As it often happens that theſe three moſt uſeful inſtruments are badly divided, either through the fault of patterns erroneous in their firſt conſtruction, or worn out by much uſe, or by the ignorance or neglect of the divider, it was thought that rules and Tables for graduating them accurately, and examining thoſe which have been already made, might be very acceptable both to the workman and young mathematician deſirous of projecting them, or to any perſon who is obliged to uſe them.

The foundation of theſe, and indeed of moſt other ſcales, is a line of equal parts, ſo ſubdivided as that one of the leaſt of theſe parts ſhall ſcarce take up a viſible ſpace, as into thouſand parts of an inch, or even more *minute*; ſo that an error of one unit in the laſt place, in taking off any diſtance, may not affect the graduations of your intended inſtrument. Furniſhed with ſuch a ſcale, and a beam compaſs with a regulating ſcrew at one end, you may lay down any number, of four, or even five places of figures, with great exactneſs.

The beſt way of dividing a right line into any poſſible number of parts, and that which is practiſed by our beſt dividers of mathematical inſtruments, is by help of diagonal lines; both becauſe the ſmalleſt diagonal ſubdiviſions are as perceptible on ſuch a ſcale as the largeſt, and that this is the moſt ſimple method of ſubdividing, and may be extended further than any other way yet known.

In order to have an exact diagonal ſcale, it is abſolutely neceſſary that the parallel lines, through which the diagonal lines are drawn, be equidiſtant among themſelves; that the diagonals be ſo likewiſe; and that, in the two outermoſt parallels in which the diagonals terminate, the ends of each diagonal muſt be juſt even with the oppoſite ends of thoſe next to it on each ſide; that is, in a line perpendicular to the parallels.

In one inch, 25 diagonal lines may be very eaſily placed, and conſequently 50 in two inches: and if theſe diagonals run through 20 equidiſtant parallel lines, they will divide every two inches of length of the diagonal ſcale into 1000 equal parts, which will be quite ſufficient for graduating the lines of Rhumbs, Chords, Sines, Tangents, and Secants, on common plain ſcales, where the radius (or 60 degrees of the Chords) takes up only two inches in length of the whole line: and to that line of Chords, the Rhumbs, Sines, Tangents, and Secants, are adapted.

There is generally another line of Chords of 3 inches radius on theſe ſcales, at the right-hand end; and juſt above that line is a line marked *M. Lon.* for ſhewing how many geographical miles are contained in a degree of longitude in any given parallel of latitude. To lay down theſe two lines, there muſt be a diagonal ſcale containing 1000 parts in 3 inches; and theſe parts or diviſions muſt be continued on to 1420, which will be ſufficient for the purpoſe, as the whole length of each of theſe two lines takes up only 1414.21 parts.

But, for laying down the above-mentioned lines of Rhumbs, Sines, Chords, Tangents, and Secants; where the radius 2 inches contains 1000 equal parts, and the length of the Tangent line (in which there are only 80 degrees laid down) is $11\frac{2}{10}$ inches, the diagonal parts or equal diviſions muſt be carried to 5760; of which, 5671.28 will be equal to the Tangent of 80 degrees, and 5758.77 to the Secant of the ſame number of degrees; as ſhewn by the Tables of Natural Tangents and Secants.

For Gunter's lines of Numbers, Rhumbs, Sines, Verſed Sines, Tangents, and Meridional Parts, every one of which takes up $22\frac{6}{10}$ inches of his two foot ſcale, a diagonal ſcale muſt be firſt made, in which $22\frac{6}{10}$ inches ſhall contain 2000 equal parts: and, for this purpoſe, there will be no need for more than 10 equidiſtant parallel lines along the ſcale, and one inch of its length will not include quite 8 equidiſtant diagonal lines.

For Sectors, of whatever length the radius of the Chords (or of 60 degrees) be, which is generally very near the whole length of the Sector when ſhut, a diagonal ſcale muſt be prepared, in which *that* length ſhall contain 1000 equal parts.—The diſtance between theſe long parallels ſhould not be leſs than a tenth part of an inch: otherwiſe the eye cannot well eſtimate the decimal parts expreſſed in the Tables, which are hundredth parts of the ſpaces between the long parallel lines, in all except the Dialing Tables of Latitudes, Hours, and Inclination of Meridians, and in the Table of Gunter's Meridional Parts; in which, the ſpaces between the parallels are ſuppoſed to contain only ten parts each.

Being thus provided with proper diagonal ſcales, patterns for graduating all the lines on Scales and Sectors may be made in the following manner.

Having drawn the lines on your intended braſs pattern, analogous to thoſe which muſt contain the graduations on ſcales, fix your diagonal ſcale cloſe by the upper edge of the pattern to be graduated: and then applying one ſide of a ſquare along the upper edge of the diagonal ſcale, and the other ſide directly acroſs both that ſcale and the pattern, cut the firſt diviſion of each line on the pattern directly even with the very beginning of the diagonals;

and each diviſion after *that*, againſt the ſame number of parts, found among the diagonals, as anſwer to the number of parts in the Tables belonging to each degree in the reſpective lines: and, where the lines are long enough to admit of the quarters of degrees, theſe may be put in, according to the numbers in the Tables——But, in the common plain ſcales, where the length of the radius of the Chord line is only two inches, the ſpaces taken up by the degrees are too ſmall to allow of ſubdiviſions, except in the Tangents and Secants after the 50th degree.

Thus, in the Tables of Natural Chords, Sines, Tangents, and Secants, againſt the firſt degree is 17.45 parts both for the Chords and Sines, and 17.46 for the firſt degree of the Tangents. Therefore, finding thoſe parts among the diagonals, and laying the croſs edge of the ſquare to them, cut the firſt degree of theſe lines on the pattern, cloſe by the ſaid edge of the ſquare: and proceed on in the ſame manner with all the reſt of the degrees in the ſeveral lines, cutting them right againſt the ſame number of parts found among the diagonals which the Tables ſhew to belong to them reſpectively. Thus, the 20th degree in the line of Chords muſt be cut even with 347.29 parts among the diagonals; the like degree of the Sines even with 342.02 parts; and the ſame degree of the Tangents even with 363.97 parts in the diagonal ſcale.

Where the Sines end on the plain ſcale, the Secants begin; namely, even with 1000 parts among the diagonals. But the Secant degrees are ſo ſmall at firſt, that there can be no putting in any of them leſs than the 10th, which anſwers to 1015.43 parts among the diagonals; and the next that can be put in is the 15th, which anſwers to 1035.28 parts: after which, all the degrees as far as 80 may be put into the line of Secants, cutting them even with the ſame numbers of parts found among the diagonals as ſtand againſt them in the Table of Natural Secants.

On Sectors, there are two Tangent lines on each leg. The firſt of theſe goes only to 45 degrees, the length of the leg admitting of no more: and the others begin at 45 degrees, at a fourth part of the length of the leg from the center of the joint to that part near the end of the leg where the 45th degree of the former line ſtands, and goes on generally to 76 degrees. The firſt of theſe which goes from 0 degeees to 45, is called the *lower Tangents*; and the laſt, which goes from 45 to 76, is called the *upper Tangents.*

The lower Tangent degrees are laid down by the diagonal ſcale according to the numbers found againſt them in the Table of Natural Tangents; 1000 parts of the diagonal ſcale taking up juſt as much length as the whole 45 degrees of theſe Tangents do. But, as the radius of the upper Tangents is only a fourth part of the length of that of the lower ones, in order to lay them down on the Sector-pattern, all the numbers in the Table of Natural Tangents above 45 degrees muſt be divided by 4, and their quotients ſought for, among the diagonals, for laying down the reſpective degrees of theſe Tangents. To ſave the operator this trouble, I have taken it, and made the following Table, which conſiſts of theſe quotients, and conſequently contains the number of parts (to be found among the diagonals) for dividing the line of upper Tangents, the 76th degree of which is even with 1002.69 parts of the diagonal ſcale.

Supplemental Table, for laying down the Line of upper Tangents on Patterns for dividing the Lines on Sectors.

Deg.	Parts.	Deg.	Parts.	Deg.	Parts.	Deg.	Parts.
45	250.00	¾	328.77	½	441.87	¼	626.33
	252.19	53	331.76		446.41		634.46
	254.40		334.79	61	451.01		642.87
	256.63		337.85		455.69	69	651.27
46	258.88		340.36		460.44		659.86
	261.15	54	344.09		465.27		668.65
	263.44		347.27	62	470.18		677.65
	265.51		350.49		475.17	70	686.87
47	268.09		353.74		480.24		696.31
	270.45	55	357.04		485.40		705.98
	272.83		360.37	63	490.65		715.89
	275.23		363.75		495.99	71	726.05
48	277.73		367.17		501.42		736.47
	280.10	56	370.64		506.95		747.17
	282.57		374.15	64	512.57		758.15
	285.07		377.71		518.30	72	769.42
49	287.59		381.31		524.13		781.00
	290.14	57	384.96		530.07		792.70
	292.71		388.67	65	536.13		805.13
	295.31		392.42		542.29	73	817.71
50	297.94		396.22		548.57		830.66
	300.59	58	400.08		554.98		843.98
	303.27		403.99	66	561.51		857.71
	305.98		407.96		568.17	74	871.85
51	308.72		411.99		574.96		886.43
	311.49	59	416.07		581.89		901.47
	314.29		420.21	67	588.96		916.99
	317.12		424.41		596.18	75	933.01
52	319.98		428.68		603.55		949.57
	322.88	60	433.01		611.08		966.68
	325.81		437.41	68	618.77		984.38

The diagonal ſcale remaining where it did, for dividing the line of lower Tangents, [*See the Remark farther on*] cut the 45th degree by the ſquare againſt 250 parts found among the diagonals, having put the croſs ſide of the ſquare to it: ſo this ſecond line of Tangents ſhall begin juſt at a fourth part of the length of the former line from the center of the joint; and then proceed with the reſt of the degrees from 45 to 76, cutting them in the pattern even with the ſame number of parts found among the diagonals as belong to them in the preceding Table.

The Secants on the Sector begin alſo at a fourth part of the length of the leg from the center of the joint; and therefore, to lay down the degrees of the Secants on the Sector pattern, all the numbers in the Table of Natural Secants from 0 to $75\frac{1}{2}$ muſt be divided by 4, and their quotients taken among the parts on the diagonal ſcale. The following Table, for this purpoſe, is the quotients of the numbers in the Table of Natural Secants divided by 4.

The Secants of $75\frac{3}{4}$ degrees would reach to 1015.55, which is 15.55 parts more than the diagonal ſcale contains.

Supplemental Table, for dividing the Line of Secants on Sector Patterns.

Deg.	Parts.	Deg.	Parts.	Deg.	Parts.	Deg.	Parts.
0	250.00		315.12		364.85		433.17
10	253.86	38	317.25	47	366.57	55	435.86
15	258.82		319.44		368.30		438.60
16	260.07	39	321.69		370.05		441.38
17	261.42		323.99		371.82		444.20
18	262.86	40	326.25	48	373.62	56	447.07
19	264.40		327.55		375.44		449.99
20	266.04		328.77		377.29		452.95
21	267.78		330.00		379.16		455.96
22	269.63	41	331.25	49	381.06	57	459.02
23	271.59		332.52		382.99		462.13
24	273.66		333.80		384.95		465.29
25	275.84		335.09		386.92		468.50
26	278.15	42	336.41	50	388.93	58	471.77
27	280.58		337.74		390.79		475.09
28	283.14		339.08		393.03		478.47
29	285.84		340.45		395.13		481.90
30	288.67	43	341.83	51	397.23	59	485.40
	290.15		343.23		399.41		488.95
31	291.66		344.64		401.60		492.57
	293.21		346.08		403.81		496.25
32	294.79	44	347.54	52	406.07	60	500.00
	296.42		349.01		408.35		503.81
33	298.09		350.51		410.67		507.69
	299.80		352.02		413.02		511.64
34	301.55	45	353.55	53	415.41	61	515.67
	303.35		355.10		417.83		519.76
35	305.19		356.68		420.29		523.93
	307.08		358.25		422.79		528.18
36	309.02	46	359.89	54	425.32	62	532.51
	311.00		361.52		427.90		536.92
37	313.03		363.18		430.51		541.42

The Supplemental Table concluded.

Deg.	Parts.	Deg.	Parts.	Deg.	Parts.	Deg.	Parts.
	546.00	66	614.62		705.63		831.38
63	550.67		620.74		713.86		843.05
	555.42		626.96		722.30	73	855.07
	560.29		633.32	70	730.95		867.46
	565.24	67	639.82		739.83		880.23
64	570.29		646.48		748.93		893.40
	575.45		653.28		758.29	74	906.99
	580.70		660.24	71	767.89		921.01
	586.07	68	667.35		777.75		935.49
65	591.55		674.66		787.89		950.46
	597.14		682.12		798.30	75	965.92
	602.85		689.77	72	809.02		981.93
	608.69	69	697.61		820.01		998.48

On Sectors, the line of Lines (which is divided into equal parts) and the lines of Chords, Sines, Tangents, and Secants, are laid down on both the legs. They are all drawn from the center of the joint, and ought to be ſtrictly at equal angular diſtances from each other, at the other ends of the legs. So that, whether the Sector be open or ſhut, the ſame opening of the compaſſes that reaches croſs-wiſe from 10 on the line of Lines on one leg to 10 on the other (at the ends furtheſt from the joint) ſhould reach from 60 to 60 degrees of the Chords, from 90 to 90 of the Sines, from 45 to 45 of the lower Tangents, from end to end of the upper Tangents, and likewiſe from end to end of the lines of Secants. I generally find all theſe very well laid down, except the lines of upper Tangents and Secants; which, for want of this precaution, are troubleſome to uſe on moſt Sectors.

I apprehend that Sectors would be much more convenient than they now are, if their lines of Chords contained all the degrees of 0 to 90. For then, in laying down an angle of any number of degrees leſs than 90, one opening of the compaſſes would do; whereas, as they now are, it requires two operations to lay down an angle of any number of degrees above 60. And beſides, if the line of Chords contained all the 90 degrees, the lower Tangents, inſtead of ending at 45 degrees,

grees, would go on to 55, by carrying them out a very ſmall ſpace beyond the end of the Chords: and alſo, the line prolonged on which the Sines are laid down (they going no further than 60 degrees of the Chords) would receive thereon all the Secants as far as 45 degrees; ſo that, all theſe Tangents and Secants might be taken off without a ſecond opening of the Sector, as is cuſtomary in common ones; which would be a very great convenience to thoſe who uſe it. And then, beginning the line of upper Tangents at 55 degrees, and of upper Secants at 55, with a fourth part of the Tabular numbers from the center of the joint, both Tangents and Secants might be carried on to 80 degrees.—For theſe purpoſes, the diagonal ſcale muſt be ſo divided, as that 1414.2 of its equal parts ſhall be equal in length to the whole line of Chords; and then, 1440 of theſe parts would extend but a very little further. And the line of Lines (which is a line of equal parts) muſt be ſo divided, as that ten of its grand diviſions, to which the numeral figures are ſet, ſhall be preciſely equal in length to 60 degrees of the line of Chords.—In common Sectors, 6 inches long when ſhut, each grand diviſion of the line of Lines is ſubdivided into 20 equal parts, every one of which is ſuppoſed to be ſubdivided into 5; by which means, the 10 grand diviſions of that line are ſuppoſed to contain 1000 equal parts, *viz.* the tabular number anſwering to the Radius, or 60 degrees of the Chords.

Nº	Parts.
4	1414.21
5	1175.57
6	1000.00
7	867.89
8	765.36
9	684.04
10	618.03
11	563.47
12	517.63

The annexed ſmall Table is for laying down the line of Polygons on Sectors where the line of Chords goes on to 90 degrees. Thus the figure (or number) 4 muſt ſtand even with 1414.21 parts of the diagonal ſcale; the figure 5 againſt 1175.57 parts; the figure 6 againſt 1000.00 parts; and ſo on to 12, as in the Table.—But, by theſe numbers, the line of Polygons could be laid down only from 6 to 12 on common Sectors, where the line of Chords goes no further than 60 degrees.

But, for thoſe who chuſe to make Sectors in the common way, the here-annexed Table ſhews the numbers in the diagonal ſcale by which the line of Polygons is to be laid down. Thus, the figure 4 muſt anſwer to 1000.00 parts of the diagonal ſcale, the figure 5 to 831.25 parts, the figure 6 to 707.11 parts, the figure 7 to 613.69, and ſo on to the laſt diviſion 12.

N°	Parts.
4	1000.00
5	831.25
6	707.11
7	613.69
8	541.19
9	483.69
10	437.01
11	402.73
12	366.02

REMARK.

As all thoſe lines which are properly called *Sectoral Lines* *, terminate in an arch whoſe center is the center of the joint, the diagonal ſcale ought to be ſo placed, as that the long parallel lines upon it may be ſtrictly parallel to each ſectoral line on the pattern to be divided from the ſcale: and alſo, that when one ſide of the ſquare is laid cloſe to the upper edge of the diagonal ſcale, and the other ſide of the ſquare (that lies acroſs the ſcale and pattern) to the center of the joint, *that* ſide of the ſquare may then be at the beginning of the diagonal diviſions on the ſcale. Then all the diviſions cut by that ſide of the ſquare will be true, and each diviſion at right angles to its own reſpective line. Without this precaution, the innermoſt ſectoral lines would not be divided to their whole proper lengths: and ſo they would not all have the ſame radius, and conſequently the meaſures taken off from them by the compaſſes would not agree together.

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* The lines which are drawn from the center of the joint almoſt to the other ends of the legs.

But, when the Sector pattern is truly divided according to this method, it may be applied cloſe to the ſide of the Sector to be divided from it; becauſe, as the lines on the intended Sector will be parallel to the like lines on the pattern, one ſide of a ſquare may be applied to the upper edge of the pattern, and the other ſide will lie acroſs both the pattern and Sector at right angles: and then, by applying that ſide to each diviſion of the pattern, and cutting each ſuch diviſion cloſe by it on the Sector, all the diviſions on the Sector lines will be true, although they be not cut at right angles to thoſe lines to which they belong reſpectively.

As the dialing lines of Latitudes, Hours, and Inclinations of Meridians, have no dependence on the radius of any of the above-mentioned lines, and are indiſcriminately put upon Sectors and plain Scales, they may be made of any convenient length where there is ſufficient room. But, as they depend upon one another, they muſt be all laid down from one ſcale of equal parts. The lines of Hours and Inclination of Meridians are of equal length, which ought to be ſix inches at leaſt; and the length of the line of Latitudes is equal to that of four hours and an half, in the line of Hours.

To lay down theſe lines, you muſt have a diagonal ſcale of ſuch a length, as that 1414.2 of its equal parts ſhall contain as great a length as the line of Hours is intended to be of. And then, the ſame number of parts, which ſtand in the Tables againſt the degrees of Latitudes, Inclination of Meridians, Hours and parts of an Hour, muſt be found among the diagonals; and the reſpective diviſions in the lines cut by the ſide of a ſquare applied to theſe parts in the diagonal ſcale, after it has been fixed cloſe to the edge of that on which theſe lines are to be divided.—Thus, 10 degrees in the line of Latitudes (reckoned from the beginning thereof, which muſt be even with the beginning of the diagonal diviſions) muſt ſtand even with 241.9 parts among the diagonals: 10 degrees of the line of Inclination of Meridians muſt be even with 212 parts of the diagonals, the hour of I againſt 298.8 parts; and ſo on, as in the Tables.

Gunter's Lines, of Rhumbs, Numbers, Sines, Verſed Sines, Tangents, and Meridional Parts (on the ſcale that goes by his name) are all laid down by one diagonal ſcale of equal parts; and 2000 of theſe parts muſt include a length equal to the whole length of the line of Numbers, which conſiſts of 18 grand diviſions of different lengths, marked with the numeral figures 1, 2, 3, 4, 5, 6, 7, 8, 9, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10; the firſt grand diviſion being the ſpace between the firſt 1 and the firſt 2, and the laſt between the ſecond 9 and the 10.—On this ſcale, the grand diviſions from the firſt 1 to the ſecond 1 are generally ſubdivided into 10 parts each, altho' they might bear four times that number from 1 to 3; 20 diviſions each from the firſt figure 3 to the figure 7, and after that, only 10 diviſions each, to the ſecond figure 1, which is at the middle of the line. The grand diviſions in the other half of the line are of the ſame lengths with thoſe in the former half; but in the latter half of the line, each grand diviſion from the figure 1 to 3 is ſubdivided into 50 parts; from 3 to 7, into 20 parts each; and from 7 to 10 at the end of the line, each grand diviſion is alſo ſubdivided into 20 parts only, on account of the ſhortneſs of the ſpaces.

Being provided with a diagonal ſcale of 2000 equal parts, which include a length equal to the intended length of the line of Numbers, fix the lower edge of it cloſe to the upper edge of the ſcale intended to be divided, and applying one ſide of the ſquare to the upper edge of the diagonal ſcale, and the other ſide to the beginning of the diagonal diviſions, cut the firſt croſs line in the line of Numbers (where the firſt 1 is to ſtand) cloſe by that ſide of the ſquare; and then, moving the ſquare onward till the ſame ſide of it comes to the number of parts among the diagonals which anſwer in the Table of *Gunter*'s Numbers to the intended ſubdiviſions between 1 and 2, cut theſe diviſions accordingly, in the line of Numbers, cloſe by the ſide of the ſquare which was ſet to the parts in the diagonals anſwering to theſe ſubdiviſions; and ſo on till the whole line be divided.

Thus, the Tables ſhew, that the figure 2, at the end of the firſt grand diviſion (marked in the Table 2.00) muſt anſwer to 301.03 parts found among the diagonals; the diviſion-line for the figure 3 muſt anſwer to (or ſtand even with) 477.12 parts, the diviſion-line for 4 againſt 602 60 parts; and ſo on, to the end of the firſt half of the line, where the ſecond figure 1 ſtands againſt 1000. The other half of the line is divided the ſame way, by the other 1000 diagonal parts of the ſcale.—The ſubdiviſions which the operator chuſes to put into this line muſt be cut even with the like number of parts found among the diagonals as anſwer to them in the Table.

In order to divide the lines of Rhumbs, Sines, Verſed Sines, Tangents, and Meridional Parts, on *Gunter*'s Scale, the diagonal ſcale muſt be placed the contrary way to what it was for dividing the line of Numbers, becauſe all theſe lines are divided backward, or from the right hand toward the left. Therefore, turn the diagonal ſcale, and placing its contrary edge to the upper edge of the ſcale to be divided, fix it ſo, as that, when the ſquare is applied, the beginning of the diagonal diviſions ſhall ſtand juſt even with the end of the line of Numbers: and then, applying the croſs ſide of the ſquare to the ſame number of parts among the diagonals which anſwer to the degrees, half degrees, *&c.* in the Tables of Rhumbs, Sines, Verſed Sines, Tangents, and Meridional Parts, cut theſe diviſions in the proper lines cloſe by that ſide of the ſquare.

As Tables of this kind were never all in print before (at leaſt ſo far as I ever heard of) and they ſerve not only for dividing the lines accurately on Scales and Sectors, but alſo for examining and proving whether the Scales and Sectors which are ſold at ſhops be accurately divided or not, I hope they will be acceptable, not only to the makers of theſe inſtruments in

general,

general, but alſo to thoſe who uſe them.

How to examine the Diviſions of the Lines on Sectors and Scales.

If the line of Lines (which is a line of equal parts) on the Sector be accurately divided, which may be eaſily tried with a pair of compaſſes, it will ſerve for examining all the other lines which are drawn from the center of the joint; for all their diviſions ought to anſwer to the equal parts of that line, as they do to the like equal parts of a diagonal ſcale from which they are ſuppoſed to be laid down according to the preceding directions.

Whatever the length of the Sector be, if the line of Chords upon it goes no further than 60 degrees, the line of Lines contains 10 grand diviſions, marked 1, 2, 3, 4, 5, 6, 7, 8, 9, 10: but if the Chords go on to 90, the line of

Lines ought to contain 1414.2 equal parts, ſuppoſing each grand diviſion to be ſubdivided into 100.

On Sectors ſix inches long, each grand diviſion of this line is ſubdivided into 20 equal parts; and if each of theſe parts be ſuppoſed to be ſubdivided into 5 (which is left to be eſtimated by the eye), each grand diviſion will contain 100 parts; and conſequently the whole ten will contain 1000.

On Sectors 12 inches long, theſe grand diviſions are ſeldom ſubdivided into more than 20 parts each, altho' there might very well be 50, and then each part could be eaſily divided into two by eſtimation of the eye; and conſequently the whole line would contain 1000 equal parts.

This being underſtood, ſet one foot of the compaſſes in the center of the joint, and extending the other foot to the ſame number of parts in the line of Lines as agree with the tabular number of parts for any degree of the Chords, Sines, Tangents, or Secants, turn *that* foot to the line of degrees you want to examine; and if it falls into the degrees anſwering reſpectively to the tabular numbers, the line is truly divided; otherwiſe not.

But if the Sector be too long for the points of the compaſſes to take in the whole length of the line of Lines between them, open it ſo far, as that the compaſſes may reach conveniently acroſs the Sector from 10 in the line of Lines on one leg to 10 in the line of Lines on the other: and then, taking the tabular numbers of parts in the compaſſes acroſs the Sector, in each line of Lines which anſwer to the tabular number of parts for each degree of the line of Lines, Chords, Tangents, or Secants, apply that extent acroſs the Sector to the like degrees of theſe lines: and if the points of the compaſſes fall directly into theſe graduations, the lines are accurately divided; otherwiſe not.

To examine the lines on plain Scales or Gunter's Scales, open the Sector ſo, as that the length of the radius of the Chords, or whole length of the line of Lines, taken in the compaſſes, their points ſhall then reach from 10 in one line of Lines on the Sector to 10 on the other: and then, taking the tabular numbers for the degrees of the Chords, Sines, or Tangents, acroſs the Sector from one line of Lines to the other, ſet one foot of the compaſſes in the beginning of the line to be examined, and the other foot forward among the degrees of that line; and if it falls juſt into each given degree, the line is truly divided; but falſely if it does not.

Thus you may examine the whole line of Lines, the line of Chords to 60 degrees, and the Tangents to 45: but, in order to examine the degrees of the Tangents above 45, and all the degrees of the Secants, you muſt take the following method.

The Sector remaining at the ſame opening as before, take the ſupplemental tabular numbers in your compaſſes acroſs the Sector from one line of Lines to the other, which anſwer to the reſpective degrees of the Tangents and Secants, and apply that extent acroſs the Sector to the like degrees in theſe lines: if the compaſs-points fall exactly into them, the lines are truly graduated.

For Gunter's lines of Numbers, Rhumbs, Sines, Verſed Sines, Tangents, and Meridional Parts, ſet one foot of the compaſſes in the beginning of the line of Numbers (at the firſt figure 1) and open the compaſſes till the other foot falls into the ſecond 1 at the middle of the line. Then open the Sector ſo, as that the ſame opening of the compaſſes ſhall reach from 10 in the line of Lines on one leg to 10 in the like line on the other. Then, taking the tabular numbers in your compaſſes acroſs the Sector in theſe two laſt-mentioned lines, which anſwer to the diviſions of the lines you want to examine, apply that extent forward from the left-hand end of the line of Numbers, but backward from the right-hand end of the other lines; and if theſe diviſions or graduations are found by the compaſſes to agree with the tabular numbers belonging to them, the lines are truly divided; otherwiſe not.

N. B. In the Sectoral Lines, which proceed from the center of the joint, each has three parallel ſtraight lines drawn for limiting the greater and leſſer diviſions: in each of theſe, it is the innermoſt line to which the points of the compaſſes muſt be applied; as that is the only line of the three that proceeds directly from the center of the joint.—I have dwelt the longer on this ſubject, becauſe theſe inſtruments are in the hands of every Mathematician, and it is of the utmoſt importance to have them rightly divided.

Having ſhewn how to divide and examine the Lines on Scales and Sectors, it is natural to ſuppoſe that the divider would be willing to know, not only how to examine the Tables themſelves, but alſo how to conſtruct them.

The Tables of Natural Sines, Tangents, and Secants, are copied from thoſe in Sherwin's Tables thereof.

The right Sine of an Arc is half the Chord of double that Arc. Therefore, double the Natural Sine of half the given Arc (or number of degrees) and *that* will be the Chord of the whole Arc, or number of degrees required; which, in the Tables, is carried no further than to 90.

The conſtruction of the Table of Natural Rhumbs is the ſame as that of the Chords.

The Table of Numbers is only the logarithms of theſe Numbers. The logarithm of unity (or 1) being nothing, is not inſerted in the Table: the logarithm of 1.01 (the ſame as that of

101 without the characteriſtic) is 43214, which may be put down 4.32; the logarithm of 1.02 is 86002, which may be put down 8.60; and ſo on, for laying down the line of Numbers on Gunter's Scale.

For the ſame ſcale, the Sines are the reſpective logarithmic Sines ſubtracted from radius, or the Co-ſecants leſs radius.

The Tangents (for the ſame ſcale) are the logarithmic Tangents, rejecting radius.

The Verſed Sines are double the logarithmic Secants of half the given number of degrees.

For the Meridional Parts, divide the Meridional Parts in any Table thereof by 60.

For the Sine Rhumbs, having found the degrees in every point and quarter-point of the Mariner's Compaſs, find the logarithmic Sines thereof, and take their complements arithmetical from radius, or from 8 points.

The ſame holds good for Tangent Rhumbs, remembering they extend only to 4 points on the ſcale, and that both the Rhumb and its complement ſtand on the ſame point of the line; a logarithm and its reciprocal being equally diſtant from radius on the oppoſite ſides.

By theſe rules I have made the foregoing Tables from the now common Tables of Logarithms, Sines, Tangents, Secants, and Verſed Sines, with all the care and accuracy that I poſſibly could, without regarding the time and pains required to conſtruct them.

Of the plain Scale, Sector, and Gunter's *Scale.*

The lines on the plain Scale are uſeful in moſt branches of the mathematics, and its uſe in each of them is to be found in almoſt every treatiſe of the practical mathematics.

The Sector is principally uſeful as a univerſal plain Scale, fitted to any radius within the compaſs of its opening; only obſerving that the equal Parts, Sines, Chords, and Tangents under 45 degrees, are not taken along one line, as on the plain Scale, but acroſs the Sector from any degree on one leg to the ſame degree of the ſame line on the other.

The Gunter's lines are for readily working proportions; in which, regard muſt be had to the terms, whether arithmetical or trigonometrical; that the firſt and third term may be of the ſame name, and the ſecond and fourth of the ſame name likewiſe: then, raiſing your proportion according to theſe rules, take the extent on their proper line, from the firſt term to the third, in your compaſſes; and applying one point of the compaſſes to the ſecond, the other applied to the right or left according as the fourth term is to be more or leſs than the ſecond, will reach to the fourth. Three examples will explain this.

1.

If 4 yards of cloth
coſt 18 ſhillings,
Then 32 yards
will coſt ?

2.

As radius
to the hypothenuſe 120,
So the ſine of the angle oppoſite the baſe 30° 17′
to the baſe.

3.

As the co-ſine of the latitude 51° 30′
(= the ſine of 38° 30′)
is to radius,
So is the ſine of the Sun's declin. 20° 14′
to the ſine of the Sun's amplitude.

Note, The Line of Numbers (on Gunter's Scale or on the Sector) is intended to ſupply the Table of Logarithms. Thoſe of Sines, Tangents, (and Secants, which are found in the ſame place with their Co-ſines) and the Verſed Sines, ſupply the places where their logarithms are neceſſary in calculation.

Now, in the firſt queſtion, on the Line of numbers take in your compaſſes the diſtance between 4 and 32; apply one foot thereof on the ſame line at 18, and the other will reach to 144, the ſhillings required.

In the ſecond, the diſtance between radius (or the Sine of 90°) and the Sine of 30° 17′, taken from the Line of Sines, and one foot applied to the hypothenuſe 120 on the Line of Numbers, and the other applied to the left (as the legs of a right lined and right angled triangle are leſs than the hypothenuſe) *that* foot will reach to 60$\frac{1}{2}$, the length of the baſe required.

The third is wrought wholly on the Line of Sines. The diſtance between the Sines of 38° 30′ and 20° 14′ taken in your compaſſes, ſet one foot on the radius or Sine of 90°, and the other will reach to 33°$\frac{1}{4}$, the Sun's amplitude required.

In the ſame manner are uſed the Tangents, Secants, and Verſed Sines, in proportions where they are required: though ſometimes the Verſed Sine is taken when the other foot ſtands on the Line of Sines, as in finding the azimuth, &c. which is eaſily performed when the art of raiſing the proportion is known.

A ſhort Account of the Logarithms.

The invention of Logarithms, ſays the great Dr. Halley, is juſtly eſteemed one of the moſt uſeful diſcoveries in the art of numbers. This aſſertion is corroborated by the united teſtimonies of every learned mathematician who has made mention of them, from the time of their firſt publication to this day.

Lord Nepair, of Merchiſton in Scotland, in the year 1614, publiſhed the firſt ſpecimen of theſe uſeful numbers, under the title of *Mirifici Logarithmorum Canonis Deſcriptio*; a book which was received with tranſport in the mathematical world. And tho' the author had reſerved the method of conſtructing his Table, till the ſenſe of the learned upon his invention ſhould be known; yet Kepler, Speidell, and others, abroad and at home, laboured at the computation of Logarithms, and conſtructed ſmall tables thereof, conformable to the plan of Lord Nepair.

The deſcription and uſe of the Canon being in Latin, induced Mr. Edward Wright, a learned mathematician of thoſe times, and to whom we owe the principles of that falſely called Mercator's Sailing, to tranſlate it for the benefit of ſuch of his pupils and others, who not underſtanding the original, were yet deſirous of being acquainted with ſo valuable a performance. This he effected with great care; but unhappily dying before the publication, that office devolved to his ſon, Samuel Wright, who, with the aſſiſtance of Mr. Briggs, then Geometry Profeſſor at Greſham College, and whoſe name ſhines with great luſtre in a hiſtory of Logarithms, publiſhed it in the year 1616 or 1618 *.

This tranſlation was dedicated by the publiſher to the Eaſt India Company, who, it ſeems, had employed Mr. Edward Wright in mathematical affairs: in which dedication he ſays, "his father's care to make the tranſ-"lation bear a true reſemblance of "the original was ſo great, that he "procured the *author*'s peruſal of it; "who, after *great pains taken therein*, "gave approbation to it. And it is "apparent enough that he (Edward "Wright) eſteemed it, and intended "to have recommended it as a book "of more than ordinary worth, eſpe-"cially to ſeamen. But ſhortly after "he had it returned out of Scotland, "it pleaſed God to call him away,

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* My copy is printed at London for Simon Waterſon, 1618.

" before he could publiſh it." Thus we ſee that the edition of Edward Wright had ſome advantages over the firſt edition, from the reviſal of the inventor, who alſo wrote a preface to the ſame; which, as it ſets forth the purſuits that led to this diſcovery, and as the book is very rare, I ſhall inſert in the words of its noble author, together with a ſpecimen and deſcription of the work itſelf.

" The Author's Preface to the *admirable* " *Table of Logarithms.*

" Seeing there is nothing, right " well beloved ſtudents in the mathe" matics, that is ſo troubleſome to " mathematical practice, nor that doth " more moleſt and hinder calculators, " than the multiplications, diviſions, " ſquare and cubical extractions of " great numbers, which, beſides the " tedious expence of time, are for the " moſt part ſubject to many ſlippery " errors, I began therefore to conſider

" in my mind, by what certain and
" ready art I might remove thoſe hin-
" drances; and having thought upon
" many things to this purpoſe, I found
" at length ſome excellent brief rules,
" to be treated of (perhaps) hereafter.
" But amongſt all, none more profit-
" able than this, which, together with
" the hard and tedious multiplica-
" tions, diviſions, and extractions of
" roots, doth alſo caſt away from the
" work itſelf even the very numbers
" themſelves that are to be multi-
" plied, divided, and reſolved into
" roots, and putteth other numbers
" in their places, which perform as
" much as they can do, only by ad-
" dition and ſubtraction, diviſion by
" 2 or diviſion by 3: which ſecret in-
" vention being, as all other good
" things are, ſo much the better as it
" ſhall be the more common, I thought
" good heretofore to ſet forth in *Latin*
" for the public uſe of mathemati-
" cians. But now, ſome of our coun-

" trymen in this Iſland, well affected
" to theſe ſtudies, and the more pub-
" lic good, procured a moſt learned
" mathematician to tranſlate the ſame
" into our vulgar Engliſh tongue;
" who, after he had finiſhed it, ſent
" the copy of it to me, to be ſeen and
" conſidered of by myſelf. I having
" moſt willingly and gladly done the
" ſame, find it to be moſt exact, and
" preciſely conformable to my mind,
" and the original. Therefore, it may
" pleaſe you who are inclined to theſe
" ſtudies, to receive it from me and
" the tranſlator with as much good-
" will as we recommend it unto you.
" Fare ye well."

And in his dedication to Prince Charles (afterwards King Charles I.) he ſays, " This his new invention
" doth clean take away all the diffi-
" culty that heretofore hath been in
" mathematical calculation; and is ſo
" fitted to help the weakneſs of me-
" mory, that, by means thereof, it is

" eaſy to reſolve more mathematical
" queſtions in one hour's ſpace, than
" otherwiſe by that wonted and commonly received manner of Sines,
" Tangents, and Secants, can be done
" even in a whole day."

The following is a ſpecimen of the firſt page of the Canon, in which I have, however, corrected the typographical errors.

Deg. 0.

Min.	Sines.	Logarith.	Differen.	Logar.	Sines.	
0	0	Infinite.	Infinite.	.0	1000000.0	60
1	291	8142568	8142568	.1	1000000.0	59
2	582	7449421	7449421	.2	999999.8	58
3	873	7043956	7043956	.4	999999.6	57
4	1164	6756275	6756274	.7	999999.3	56
5	1454	6533131	6533130	1.1	999998.9	55
6	1745	6350810	6350808	1.6	999998.6	54
7	2036	6196659	6196657	2.2	999998.7	53
8	2327	6063129	6063126	2.8	999997.4	52
9	2618	5945345	5945342	3.5	999996.7	51
10	2909	5839986	5839982	4.3	999995.9	50
11	3220	5744676	5744671	5.2	999995.0	49
12	3491	5657665	5657659	6.2	999994.0	48
13	3781	5577622	5577615	7.3	999992.8	47
14	4072	5503514	5503506	8.4	999991.7	46
15	4363	5434523	5434513	9.6	999990.5	45
16	4654	5369984	5369973	10.9	999989.2	44
17	4945	5309360	5309348	12.3	999987.8	43
18	5236	5252202	5252188	13.8	999986.3	42
19	5527	5198136	5198121	15.4	999984.7	41
20	5818	5146843	5146826	17.0	999983.1	40
21	6109	5098054	5098035	18.7	999981.3	39
22	6399	5051534	5051514	20.5	999979.5	38
23	6690	5007083	5007061	22.4	999977.6	37
24	6981	4964524	4964499	24.5	999975.6	36
25	7272	4923703	4923676	26.5	999973 6	35
26	7563	4884483	4884454	28.7	999971.4	34
27	7854	4846743	4846712	30.9	999969.2	33
28	8145	4810376	4810343	33.2	999966.8	32
29	8436	4775286	4775250	35.6	999964.4	31
30	8726	4741385	4741347	38.1	999961.9	30
						Min.

Deg. 89.

The columns marked Sines are the natural ſines anſwering to the degree and minute of the firſt and laſt columns; the adjoining ones their logarithms, and the middle one the differences between the logarithms. The Author explains every one of theſe columns; but I ſhall only inſert, in his words, what relates to thoſe containing the logarithms.

"§ 11. The 3d column containeth "the logarithms of the arches and "ſines towards the left hand.

"§ 13. And they are alſo the loga-"rithms of the complements of the "arches and ſines towards the right "hand, which we call Antilogarithms.

"§ 14. The fifth column containeth "the logarithms of the arches and "ſines towards the right hand.

"§ 16. They are alſo the antiloga-"rithms of the arches and ſines to-"wards the left hand, or the loga-"rithm of the complements.

"§ 17. Laſtly, the 4th or middle "column containeth the differences

“ between the logarithms of the 3d
“ and 5th columns; and ſo this co-
“ lumn is twofold, abounding and
“ defective.

“ § 18. Thoſe differences are abound-
“ ing which ariſe out of the ſubtrac-
“ tion of the logarithms of the fifth
“ column from the logarithms of the
“ third column.

“ § 19. But the differences ariſing
“ by ſubtraction of the logarithms of
“ the third column out of the loga-
“ rithms of the fifth column, are de-
“ fective; which therefore are leſs
“ than nothing.

“ § 20. The abounding differences
“ are called the differential numbers
“ of the arches towards the left hand.

“ § 22. And are alſo the logarithms
“ of the tangents of the left hand
“ arches.

“ § 23. But the defective differences
“ are called the differential numbers
“ of the right hand arches.

“ § 25. And are alſo the logarithms
“ of

“ of the tangents of the right hand
“ arches.”

And as *admonitions*, amongſt others, he gives the following:

“ 28. Here it is to be noted, that
“ if you make the logarithms of the
“ 3d column defective, ſetting before
“ them this mark —, they ſhall be
“ made the logarithms of the hypo-
“ thenuſes or ſecants of the right hand
“ arches of the ſeventh column.
“ 30. And if you make the loga-
“ rithms of the fifth columns defec-
“ tive, they ſhall be the logarithms of
“ the hypothenuſes, or of the ſecants
“ of the left hand arches of the firſt
“ column.”

Thus the nature of the Canon required ſome knowledge of algebraic addition and ſubtraction; and it was beſides troubleſome thereby to find the intermediate numbers; the logarithms being only given for ſuch numbers as were equal to the natural ſines of each minute. The latter inconvenience E. Wright endeavoured to remove by a ſcale, of which a draught and deſcription is given in the tranſlation of the admirable Canon, but which Mr. Briggs performed much better in the ſame work, by means of a ſubſidiary table of ſimple application.

The firſt of theſe inconveniencies was well removed by John Speidell, who was a very ingenious man, and firſt completed the Canon for trigonometrical uſe, by adding three other columns, which were the complements arithmetical of Lord Nepair's, to wit, Secant, and Co-ſecant and Cotangent leſs radius. By this means he ſaved the application of the plus and minus, and rendered the calculus of triangles more eaſy. This he publiſhed in the year 1619, under the title of *New Logarithms*; and to the ſixth impreſſion thereof in 1624, is added a Table of the Logarithms of Natural Numbers to 1000, in which the logarithm of 1 is nothing, and that of one thouſand, 690775.—Kepler too, in the year 1724, publiſhed at Marpurg his *Chilias Logarithmorum ad totidem Numeros rotundos*; in which his logarithm of 1 was 1611809.59, and of 100000, nothing; which alſo anſwered to the logarithm of 90°. And as Nepair had given the logarithms to every degree and minute of the quadrant, ſo Kepler gives the neareſt correſponding degree and minute, anſwering to each of his logarithms.

But of all that beſtowed their labour on the firſt ſort of logarithms, Benjamin Urſinius is moſt worthy of mention, who calculated a table of ſuch logarithms to every 10 ſeconds, which was printed at Cologne in 1625, and to which *Vlaac* muſt have been much obliged in conſtructing his *Canon Magnus*.

Theſe attempts ſerved to facilitate and enlarge the uſe of Nepair's firſt ſyſtem of logarithms, but were all nevertheleſs ſubject, in ſome meaſure, to its inconveniencies. The perfection of thoſe noble numbers was reſerved as an additional honour to their firſt inventor; and the preſent ſyſtem, than which there can ſcarcely be hoped for a better, is likewiſe the invention of Lord Nepair.

Mr. Henry Briggs, at the time when the *Canon Mirificus* appeared, was Geometry Profeſſor at Greſham College. This man joined to a great genius and wonderful ſagacity in mathematical affairs, a moſt indefatigable induſtry, with ſingular purity of manners, candour, and generoſity. Inflamed by this truly admirable work, he writes, on the 10th of March 1615, to Mr. (afterwards Archbiſhop) Uſher, "that "he was wholly taken up and em"ployed about the noble invention of "logarithms, then lately diſcovered." And again, "Nepair Lord of Markin"ſton hath ſet my head and hands at

" work with his new and admirable
" logarithms; I hope to ſee him this
" Summer, if it pleaſe God; for I
" never ſaw a book which pleaſed me
" better, and made me more wonder."
And we find he actually computed new logarithms, wherein he made o the ſine total, as in the Canon, but made 1000000000 the logarithm of one-tenth part of the whole ſine, or of 5 degrees 44 minutes 21 ſeconds, judging them more commodious than thoſe of Nepair; which, in the Summer of 1616, he carried with him to Edinburgh, where he was received by the Baron with great kindneſs, and ſtaid with him a month.

It was in the courſe of this time, by Mr. Briggs's account, that Lord Nepair communicated to him his intended alteration of the Canon; and that he had cauſed it to be printed in its preſent form, till his health and leiſure ſhould permit him to calculate another, wherein he intended to make o the logarithm of unity, and 10 that of the ſine total. Briggs, greatly pleaſed with that ſuggeſtion, which he confeſſed was much more eligible than his own, threw aſide thoſe he had begun, and, as well from his own inclination as the earneſt entreaty of Lord Nepair, on his return to London, ſet about the calculation of new ones, after that form: of which having computed 1000 to 8 places of figures, beſides the index, he carried them, in the Summer of 1617, to the Baron, who was highly pleaſed therewith, and preſſed his beloved friend Briggs, as he ſtiles him, to continue the work. Which injunction he afterwards more ſtrongly enforced, by the public teſtimonies he gave of both the capacity and induſtry of his friend; and the expectations the world might form from his continuing the great work, of which his Lordſhip, in regard to theſe improved logarithms, challenged little more than the invention and mode of conſtruction.

Nor were the ſentiments of Briggs in regard to Lord Nepair leſs lively than thoſe of the latter for him; he mentions him every where with the utmoſt veneration, warmth, and affection; and ſeems perfectly enraptured by the nobleneſs and utility of the great invention. And indeed nothing leſs could have ſuſtained him through the immenſe labour of ſuch computations, as he underwent in bringing it into practice.

On his return to London, he printed the *Chilias Prima*; and ſeven years after, in 1624, he produced his *Arithmetica Logarithmica*, wherein he gives the logarithms of 31000 natural numbers to 14 places of figures, beſides the index; a work which will appear ſtupendous to any one who, by the ſame method, will take the trouble to compute the logarithms of only two or three ſuch numbers; which method he may find in the ſaid work, in Malcolm's Arithmetic, Keil's Euclid, Ward's Mathematics, and ſeveral other writers; and of which an idea may be formed from the account of Dr. Halley, who ſays, that to have his logarithm true to 14 places of figures, Mr. Briggs, by continual extraction, was obliged to find the root of more than the 140 million of millioneth power. Euclid Speidell ſays, he was told, that only finding the logarithm of 2 true to 15 places, for the aforeſaid Table, was the work of eight perſons a whole year; and Pardies, in his Geometry, praiſes God, who, for the public good, has raiſed up perſons to whom he had given ſufficient patience to ſurmount the fatigue of a labour ſeemingly inſupportable: "for," ſays he, "we know that more than 20 perſons, employed for that purpoſe, "ſpent upwards of 20 years with indefatigable aſſiduity in the calculation." This laſt teſtimony, which ſome have thought an inſinuation that the logarithms were firſt invented in France, rather proves the genius and ability of our excellent countryman Briggs, who, in eight years, with ſcarce as many perſons, performed a much greater work than 20 Frenchmen in thrice the time: for the largeſt French tables of thoſe times were carried to only 11 places of figures, and theſe are ſuppoſed to have been taken from Vlaac.

Though the *Chilias Prima* was printed in the latter end of the year 1617, it was nevertheleſs not publiſhed till after the death of Nepair, which happened on the 3d of April, 1618; for, in the preface thereto, Briggs ſays, "why theſe logarithms differ from "thoſe ſet forth by their moſt illuſ-"trious inventor, of ever worſhipful "memory, in his *Canon Mirificus*, it is "to be hoped his poſthumous book "will ſhortly make appear."

In 1620, two years after the *Chilias Prima* of Briggs was publiſhed, Mr. Edmund Gunter ſet forth a Canon of Sines and Tangents, adapted to theſe logarithms, which was the firſt of that kind, and to which he gave the name of *Artificial* Sines and Tangents. This muſt have coſt Gunter ſome labour, though he appears to have been much beholden to the *Chilias*, which conſiſts of 8, and theſe only of 7 places, beſides the index, of which the laſt place is anomalous, when compared with thoſe which Briggs and Vlaac afterwards computed. Theſe he republiſhed in his book *De Sectore et Radio*, in 1623, together with the *Chilias Prima* of his old colleague.

But Briggs himſelf lived to complete a Table of logarithmic Sines and Tangents, to the one hundredth part of every degree, and to 14 places of figures, beſides the index; to which he intended to have written a full deſcription and uſe, when death put an end to his labours, at the age of 74, on the 26th January, 163^{0}_{1}, thirteen years after his beloved Nepair.

It is remarkable of theſe two great men, that they both undertook theſe mighty labours at rather an advanced age. The Compendium of Honour mentions Nepair to have died in his 67th year; and as it is generally ſuppoſed the logarithms were invented by him about 1610, he muſt have been near 60 years of age, and, as himſelf informs us, weak and infirm of body, when he began the calculation of his Canon; and in 1614, when Briggs firſt applied his thoughts that way, he muſt have been upwards of 57: which may teach us, there is no time of our life too late to acquire honeſt fame, when we are obedient to the voice of genius, and keep our eaſe in due ſubjection to induſtry.

Briggs, when dying, recommended his laſt-mentioned work to the care and completion of Henry Gellibrand, then Aſtronomy Profeſſor at Greſham College, who added thereto the deſcription and uſe of the Canon, and publiſhed it at London *, in 1633, un-

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* At Gouda, according to ſome writers.

der the title of *Trigonometria Britannica.*

Thus were theſe numbers completely planned by the wiſdom of the ingenious Lord of Merchiſton, and the great building rear'd, in nice conformance thereto, by the moſt induſtrious and learned Henry Briggs, in a few years; and it is difficult to determine in theſe two great men, which is the moſt admirable; the ſagacity of the inventor, or the indefatigable application of the calculator.

In the *Arithmetica Logarithmica*, Mr. Briggs had calculated the logarithms of all the natural numbers from 1 to 20000, and from 90000 to 101000; leaving the interval to be filled by the ingenious; to any of whom, in his preface, he offers paper properly ruled, and neceſſary inſtructions, he purpoſing, in the mean time, to employ himſelf on his Triangular Canon, and having left no labour which an ordinary ſkill might not perform. But Adrian Vlaac, of Targou, or Gouda in Holland, completed the 70000 intermediate numbers with ſuch expedition, that, in 1628, he publiſhed the ſecond edition of the *Arithmetica Logarithmica*, wherein was contained all the logarithms from 1 to 100000 to 10 places of figures, together with a Table of *Artificial* Sines, Tangents, and Secants (as Gunter had called them) to every minute of the quadrant.

Conſidering alſo that the uſual method of angular ſupputation was by minutes and ſeconds, *Vlaac*, in the ſame year with the *Trigonometria Britannica*, publiſhed his *Trigonometria Artificialis*; wherein is contained the logarithmic Sines and Tangents to every 10 ſeconds, and the logarithms from 1 to 20000: which ſhews *Vlaac* to have been a very aſſiduous and ingenious man; though Dr. Newton, Norwood, and alſo Wingate, ſeem to cenſure both him and others, who raſhly publiſhed new editions of Briggs's *Logarithmica Arithmetica* without his leave; thereby preventing the additions he intended to give it when republiſhed, and for making mutilated tranſlations of that and his *Trigonometria Britannica*, to the diſcredit as well as diſadvantage of the author. Be this as it will, *Vlaac*'s editions are in great eſteem, for having the intermediate Chiliads, for his Sexagenary Notation, and for the exactitude of his Tables.

Theſe Tables have been reduced into a more compendious form by the Reverend Nathaniel Roe, of Binacre in Suffolk, in 1633; by Dr. Newton in 1658; and the finiſhing hand ſeems to be given to them, in point of diſpoſition and fulneſs, by Sherwin, in 1706; in whoſe Mathematical Tables are contained, to 7 places of figures beſides the index, the logarithms of numbers from 1 to 100000, with natural and logarithmical Sines, Tangents, Secants, and verſed Sines, to every minute of the quadrant.

The honour of this noble and uſeful invention has been almoſt univerſally attributed to the Great Man to whom it is due; yet there have been ſome found who have, through envy, prejudice, or want of conſideration, attempted to leſſen or tranſplant it. And, firſt, Wood, in his *Athenæ Oxonienſes*, tells it as a report of Oughtred's, that one Dr. Craig firſt gave Lord Nepair an intimation that ſuch numbers were invented by Longomantanus; which ſet him about calculating his firſt Canon. Now this ſtory refutes itſelf; firſt, by Oughtred's own acknowledgment of Lord Nepair as the inventor; and by Longomantanus never claiming the invention, though he lived many years after the honour thereof was attributed to Nepair. Wingate, who carried this invention into France, in his publications, aſſerts, "the ſecond ſort of logarithms "were jointly ſchemed by Briggs and "Nepair;" contrary to the ſenſe of the very paſſage he quotes from Briggs himſelf. And, I preſume, following, or rather willing to improve upon him, Saunderſon, in his Algebra, aſſerts,

ſerts, "that Briggs was *undoubtedly* the "firſt that thought of this (the preſent) ſyſtem:" which aſſertion could be contradicted by Lord Nepair's claim in Wright's Tranſlation of his Canon, and that of his ſon for him, in the preface to his father's Poſthumous Works, 1619, both printed under the inſpection of Briggs, if even the poſitive teſtimony of Briggs himſelf were not againſt it.

And it may be obſerved of this ingenious man, that every attempt to leſſen his reputation has only ſerved to eſtabliſh or increaſe it: the trivial, and often ridiculous arguments of his opponents; the teſtimonies of his honeſt and candid coadjutors Briggs and Wright, as well as of thoſe beſt acquainted with mathematical hiſtory, foreigners as well as Britons, uniting to fix the invention of *Logarithms* in their firſt and preſent form to Lord John Nepair of Merchiſton.

The logarithmic Canon ſerves to find readily the logarithm of any aſſigned number; and we are told by Dr. Wallis, in the 2d volume of his mathematical works, that an antilogarithmic Canon, or one to find as readily the number correſponding to every logarithm, was begun by Mr. Harriot the algebraiſt (who died in 1621) and completed by Mr. Walter Warner, the editor of Harriot's works, before 1640; which ingenious performance it ſeems was loſt, for want of encouragement to publiſh it.

A ſmall ſpecimen of ſuch numbers was publiſhed in the Philoſophical Tranſactions, for the year 1714, by Mr. Long of Oxford; but it was not till 1742 that a complete antilogarithmic Canon was publiſhed, by Mr. James Dodſon, wherein he has computed the numbers correſponding to every logarithm from 1 to 100000, to eleven places of figures.

The logarithmic numbers were diſpoſed on ſtraight lines by *Gunter* in 1623, and hence the Gunter's Scale: by *Wingate*, 1627; on two rulers ſliding againſt each other, to ſave the uſe of compaſſes in working, and alſo on circles, by *Oughtred*, about 1627; in a ſpecial form by Mr. *Milburne* of Yorkſhire, about 1650; and laſtly, on the preſent ſliding Gunter, by *Seth Partridge*, in 1657.

The errors of the preſs are never more numerous than in books of Mathematics, eſpecially Tables; the calculiſt, compoſitor, examiner, and even the printer, all being liable to produce them. Warmth and precipitancy are the means which occaſion them in the former; and the printer's ink-balls will ſometimes pull out a letter or figure, which he has not always the knowledge of, and when he has, does not perhaps take the pains to repair the injury properly: a figure ſnatched up in haſte, often ſupplying, at a venture, the deficiency. From hence it ſeems next to impoſſible to be aſſured that a whole impreſſion is alike, or that it is free from error. Now, as the preſent ſeries of logarithms requires nothing but correctneſs to make it perfect, I cannot help concluding this ſhort diſcourſe with a wiſh, that our public-ſpirited noblemen and gentlemen would unite in producing a national edition of this moſt excellent BRITISH INVENTION, in a manner worthy themſelves.

If Gardiner's Logarithms, freed from errors, were deeply engraved on ſtrong copper-plates, they would endure for ages; would be free from typographical *lacunæ*; and, as the errors occaſionally found therein might be corrected by the engraver, would be continually increaſing in value. Alſo, as by national or private generoſity, in defraying part of the firſt expence, the work might be ſold reaſonable enough to come within the purchaſe of moſt mathematicians, it would be a univerſal benefit, and will immortalize his name who firſt promotes it.

FINIS.

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